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Handbook for the Manybeam Velocimeter

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Handbook for the Manybeam Velocimeter

Ted Strand
B-Division
LLNL
February 6, 2002

Outline

This handbook started out as a manila folder entitled "Useful Stuff" which I carried around with me almost wherever I went (on the job, anyway). This folder contained all kinds of different sheets of paper with equations or alignment tips or lens focal lengths--anything I though might be useful while working on the analyzer table. The number of papers in this folder continued to grow until it started to become so unmanageable that eventually I was often not able to find the piece of information I was looking for, effectively defeating the whole purpose of carrying the folder around in the first place. Clearly, some kind of organization was going to be necessary sooner or later. This inconvenience alone, however, was not enough to prompt me to start putting the information together as a handbook.

The additional impetus to create a handbook came with the hiring of a new analyzer table operator for our operations at U1a. The questions she asked as she went through the learning process were of such a fundamental nature that I began to believe some form of reference manual would be a useful tool to have. Indeed, many of her questions were the very same ones I had asked when I was learning to operate an analyzer table. Even veteran operators occasionally need to look up some detail about the analyzer table, so I wanted to put together a handbook with enough basic information to be useful to a new-hire and enough detailed information to be useful to a veteran operator.

I wrestled for quite some time over how to organize this handbook. I finally decided to "follow the light"--that is, I put the topics in the order that a pulse of light would traverse the entire system. A look at the table of contents shows the first seven topics to be, in order, lasers, fibers, probes, analyzer table, streak cameras, film, data analysis. Within the section on analyzer tables, the topics are again in the order that light travels through the table. My thinking here was that a person looking up a topic could open the handbook almost anywhere and could quickly find the topic of interest with only a basic knowledge of how light travels through the system. Of course, the table of contents may also be used. Wherever possible, I present the information as easy-to-read graphs or simplified formulas.

The final sections--8 through 10--contain other useful information. Section 8 is aimed at the novice and describes some basic optics with examples on the analyzer table (the example topics are given in parentheses). Section 9 contains a random selection of information that I find I need on occasion; some of this information I shamelessly xeroxed from my CRC Math Handbook. The last page of the handbook, section 10, contains the four equations that I use the most and nearly always need to look up. I put this page last so that it is easy to find. I also have a copy of this page tacked to the bulletin board in my office.

As you use this handbook, I would appreciate any comments you may have on other topics to include, or whether any information is incorrect or presented in an unclear manner. Thanks.

Acknowledgements

Everyone who has ever worked on the Manybeam velocimeter deserves some of the credit for the information in this handbook. The present state of the system is the culmination of many people working with it every day. While the basic optics design remains constant, the details of the hardware are continually changing as new products become available and as operators think up better ways to do things. Some of the information in this handbook will most certainly go out of date eventually. Make notes and corrections in these pages as you work with the system and the peripheral components, and pass them on to me. I will update the information as time permits.

The lion's share of the credit for the Manybeam system goes to David Goosman. He came up with the basic design for the analyzer table and most of the custom optics that make up the overall system. In particular, David conceived of the shaper and the nested lens. Both of these design concepts greatly improve the overall performance and efficiency of the Manybeam system.

Bob Druce provided information about the Continuum pulsed lasers, and Rex Avara looked up information about ordering hard film. Finally, I imposed upon Ed Daykin and Ray Eichholz of Bechtel/Nevada to read through this handbook after I had finished the first draft. Their comments about content and technical details helped improve the quality and usefulness of this handbook.

Section 1

Lasers

Our pulsed lasers are custom-built by Continuum (www.continuumlasers.com) for long pulses. They are Nd:YAG lasers which emit light at 1064 nm in the infrared. This color of light is infrared (IR), which is invisible to our eyes. Unfortunately, it is also invisible to the streak cameras, so the lasers have a doubling crystal (Second Harmonic Generator) near the output which turns some of the light to green at 532 nm (double the frequency means half the wavelength). This doubling crystal is about only 5% efficient, so 95% of the laser output, which is still IR, is sent to beamdumps inside the laser housing. Generally, we need 50 - 100 W per probe during an experiment; for 20 probes that means 1000 - 2000 W from the laser. No one knows how to build a big pulsed green laser without doing something like frequency-doubling.

The next page shows a layout of the Continuum laser and the various components (this figure was copied from the handbook for the HEAF laser). Not shown is the imbedded CW laser, which would be in the lower right hand corner. The performance of the Continuum laser varies somewhat from facility to facility, but the following numbers are good approximations.

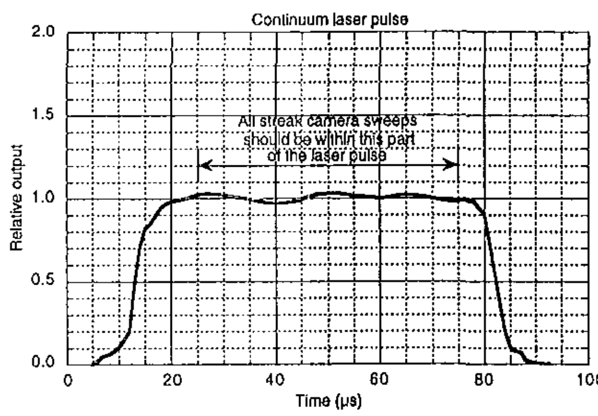
Nd:YAG (IR)

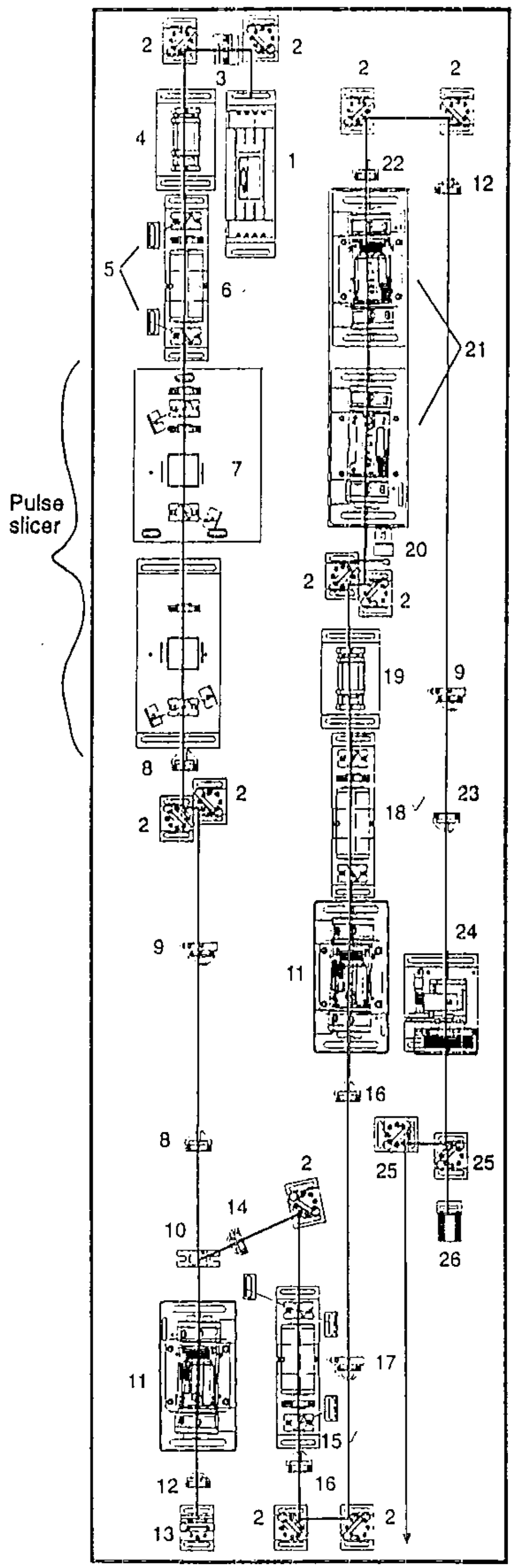
Seed laser: Lightwave Electronics
500 mW CW @ 1064 nm
Wavelength = 1064 nm
Output energy = 1 - 2 J/pulse
Pulse length = 50 to 80 μs
Linewidth < 1 MHz

Doubled (green)

Wavelength = 532.1 nm
Output: 100 mJ in 50 μs pulse
70 mJ in 80 μs pulse
Average output power = 900 - 2000 W
Amplitude variation = ± 10%
Output beam radius = 0.75 mm
Divergence (half angle) = 0.5 mrad
Emittance = 0.38 mm-mrad

The output of the pulsed laser at 532 nm should be relatively flat-topped, as shown below. This helps maintain constant fringe brightness on the streak camera records so long as the reflectivity of the experimental surface does not change. The length of the pulse must be long enough to encompass the full period of time from the beginning of the earliest camera sweep to the end of the latest camera sweep.





Legend

1. Lightwave CW
2. 45° mirrors, 105-0002
3. λ/2 plate, 108-0004
4. Assy. telescope, +155, 101-0001
   -120, 102-0007
5. Beam dumps
6. Faraday rot., 506-2320
7. PF510 pulse
   slicer
8. Lens, 1",+300mm, 101-0060
9. Pinhole, 700µm, ceramic,
   314-0259
10. Dielectric polarizer, 199-0055
11. 811U-06 head, 507-0700
    rod, 201-0056
    flashlamp, 203-0019
    "O"ring kit, 507-0710
12. λ/4 plate, 108-0001
13. Mirror, flat, 0° 1FHR, 1064,
    105-0001
14. λ/2 plate, 108-0004
15. Faraday rotator, FR 07,
    506-2310
16. Lens,+400mmx25.4 dia.,
    101-0084
17. Pinhole, .50mm, 314-0308
18. Faraday rotator, FR 09,
    506-2300
19. Assy. telescope, +155,101-0001
    & -104, 102-0005
20. Shutter, 601-0506
21. 812V-09 head, down,
    507-0900
    812V-09 head, up, 507-0900
    rod, 201-0005
    2 flashlamps, 201-0032
    "O"ring kit, 504-2050
22. Lens, +1000, 101-0069
23. Lens, +200x25.4, 101-0074
24. Lens, KTP, SHG, 5x5x20mm,
    202-0168
25. Mirror, 45°, HR1064nm,
    105-0022
26. Beam dump

There are certain steps in preparing for a shot when we need 532 nm CW light launched into the probes, rather than pulsed light. These steps include:

1) visual verification of probe alignments on the shot,
2) verification of probe assignments to streak cameras ("blink tests"),
3) verification that the numerical aperture from the return fiber is filled ("donut checks"),

and, for remote operations at U1a,

4) continuity checks ("footballs"),
5) verification of fringes on the streak camera slits just before the shot.

For these purposes, all of our Continuum pulsed lasers have a CW laser imbedded within the laser housing. This laser is shown schematically in the lower right hand corner of the sketch in Section 1.1.3. There is a moveable mirror which can be inserted into the path of the pulsed beam to allow the CW beam to exit the laser housing instead.

There are different imbedded CW lasers used at the different facilities:

HEAF	10 mW Coherent
Site 300 Bunker 851	50 mW Adlas
U1a	2 W Verdi

The Coherent laser for the HEAF facility is the same type described in Section 1.2 used on the analyzer tables. The Verdi laser at U1a has a high output to accommodate steps 4 and 5 above.

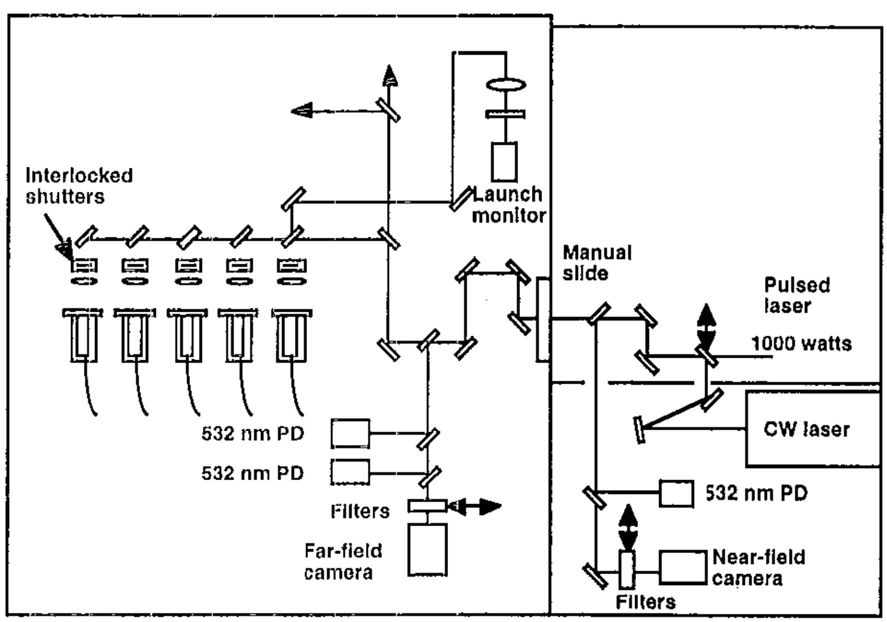
In addition to the steps given above, the laser operator uses the CW laser to align the lasers to the fiber cores in the beamfarm (Section 1.1.3). An important criteria, then, is that the CW beam be almost exactly co-linear with the pulsed beam. This assures that the pulsed laser will also be aligned properly to the fiber cores and not burn the fibers. There is a set of mirrors which adjusts the alignment of the CW beam to bring it co-linear with the pulsed beam. The figure in the beamfarm section (Section 1.1.3) shows the set of near-field and far-field cameras which are used to verify that the two beams are co-linear.

Only a single laser beam is emitted from the laser housing, whether pulsed or CW. We generally launch this beam into as many as 20 fibers for the various probes on a shot. All of our lasers send their single beam into a beamfarm, which is a series of splitters, to divide the laser beam into as many beams as there are probes on the shot. The figure below shows many of the important features in a typical beamfarm. The left side of the figure is the beamfarm, while the right side shows the outputs of the pulsed and CW lasers.

The left side of the beamfarm shows a set of five splitters used to launch the laser into five fibers. Not shown are the additional sets of five splitters. Generally, the same amount of laser power is launched into all probe fibers, so the split ratios, going from right to left in the figure, are 20%, 25%, 33%, 50% and 99%.

It is important to verify that both the pulsed and CW lasers are aimed directly into the centers of the fiber cores. Burning of the fiber endfaces can occur if the beam wanders off to the buffer. For this reason, all of our beamfarms have launch monitors, shown in the upper right corner of the beamfarm. Approximately 4% of the light launched to a fiber reflects off the endface. This reflected light is directed by a series of mirrors to the launch monitor camera. One interesting optics problem arose in first setting up the launch monitor. If the camera were focused for the CW laser, then it would be out of focus for the pulsed laser. It was determined that the gaussian beamwaists of the two lasers are different distances from the fibers. For this reason, a set of mirrors was inserted into the beamfarm to increase the optical path length to the fibers by two meters. This put the fibers into the far field for both lasers, so that the launch monitor was in focus for both lasers.

Also located in the beamfarm are photodiodes to monitor the pulse shape of the pulsed laser, as shown in Section 1.1.1.



In our analyzer rooms, we have basically one type of laser. This is a CW laser mounted on each analyzer table and is used for table alignments and producing fringes during set-ups.

The CW laser on each analyzer table is built by Coherent (www.coherentinc.com). They are diode-pumped solid state (DPSS) lasers built for 50 mW output, but are dialed down to 10 mW at the factory. An output of 10 mW is Class 3b and presents possible eye hazards, therefore, all of our CW lasers have permanently mounted filters to lower their outputs to less than 1 mW (Class II).

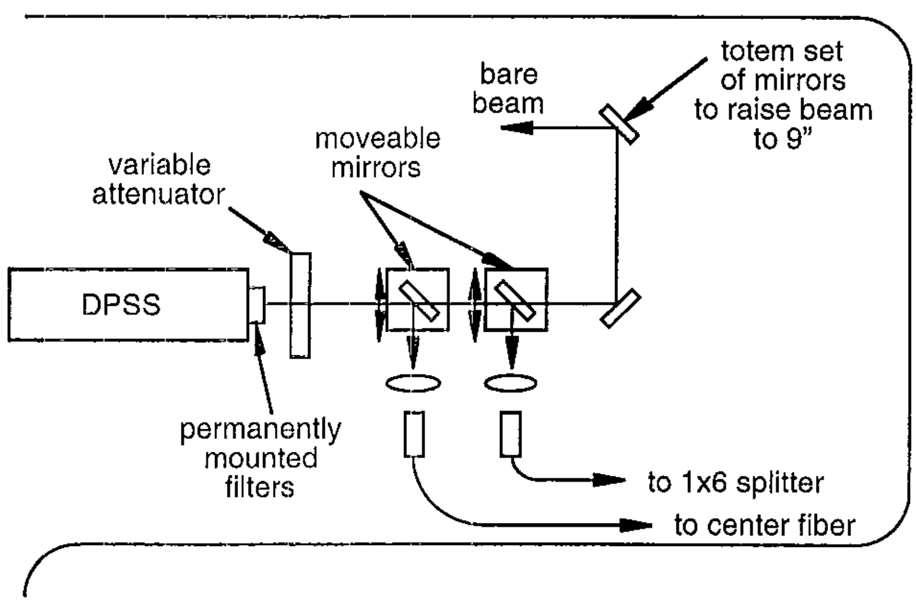
Wavelength = 532.1 nm
Power onto table < 1 mW
Beam radius = 0.35 mm
Divergence (half angle) = 0.65 mrad
Emittance = 0.2 mm-mrad

These lasers are mounted with a variable filter in front to adjust the light level during different parts of the table set-up. All of our tables have an arrangement of mirrors on linear stages to launch the laser into one of three directions:

1) Bare beam for initial alignment of the optics on the table and to retro the etalon.

2) The center fiber in the fiber chuck (Section 4.3) to check the optics and to tune the Burleigh etalons.

3) To a 1x6 splitter to launch light into five fibers to produce fringes on each streak camera.



Before you may operate the Class II CW lasers on the analyzer tables, you must have some training:

HS-5200 Laser Safety (every 5 years)
HS-5220 Electrical Hazards Awareness (every 5 years)
Laser eye exam

You should also read Chapter 28 of the Lab Health and Safety Manual. In addition, you will be required to read the appropriate FSP or OSP for the facility in which you will be operating the analyzer table.

Before you may operate any of the Class IV pulsed lasers, you must have extensive hands-on training, plus additional courses beyond those listed above.

Laser Classifications for CW lasers

Class I	very low power or imbedded, no hazards
Class II	Power (P) < 1 mW, applies to visible only
Class IIIa	$1 < P < 5$ mW visible and $0 < P < 5$ mW invisible
Class IIIb	$5 < P < 500$ mW, these can be dangerous
Class IV	$P > 500$ mW, these are dangerous

Maximum permissible exposures (MPE) for eye safety. This applies only to lasers with visible wavelengths (400 nm to 700 nm):

CW lasers	1 mW	0.25 sec aversion time
Pulsed	0.5 $\mu J/cm^2$	$1 \text{ ns} < t_{pulse} < 18\ \mu s$
	$\left(1.8 * t_{sec}^{0.75}\right) x10^{-3}\ J/cm^2$	$t_{pulse} > 18\ \mu s$

For our doubled YAG lasers with 60 μs pulse, the MPE limit is 1.23 $\mu J/cm^2$.

Safety tips:

Fibers:

- Do not look into the end of a fiber unless you are sure that there is no laser power in it.
- Inform the pulsed laser operator before cleaning fibers.
- At Site 300, you are not allowed to use fiberscopes to inspect fibers.

Lasers:

- Do not lower your head near the optical table with the laser on.
- Do not sit near any optical tables with your eyes at beam height.
- Always wear laser safety goggles in the analyzer room when the pulsed laser is operating Class IV and is sending light to the analyzer table.

Section 2

Fibers

We use two different types of fiber for the Manybeam system. The launch fiber carries high-power (50 - 100 W) light with 0.11 NA and the return fiber carries low-power (5 - 10 mW) light with 0.22 NA. The use of two fibers, rather than a single fiber probe geometry, greatly reduces the amount of undoppler-shifted light delivered to the analyzer table.

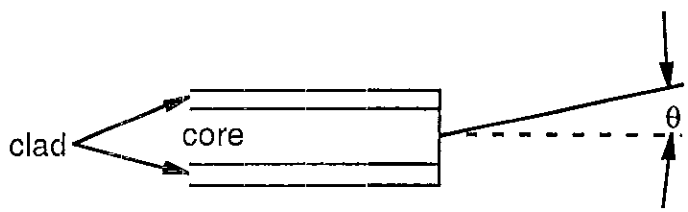
Our fibers are made by Polymicro Technologies in Phoenix, AZ. They are step-index, doped-silica clad, high-OH fiber. The 0.22 NA fiber has part number FVP100110125 and the specifications for this fiber can be found at www.polymicro.com/fvpage.htm. The 0.11 NA fiber is custom made with part number FGP100140170.

		Diameters (μm)			
Fiber	NA	Core	Clad	Buffer	1996 Price
Launch	0.11	100	140	170	$1.85/m
Return	0.22	100	110	125	$1.20/m

Definition of numerical aperture (NA)

$$NA = \sin(\theta) = \sqrt{n_{core}^2 - n_{clad}^2}$$

where θ = the half-angle of the cone of light emitted from the fiber
n_{core} = index of refraction of the core
n_{clad} = index of refraction of the clad



Note: for NA = 0.11, $\theta = 6.3°$
for NA = 0.22, $\theta = 12.7°$

The actual values of index of refraction for the core and clad at 532 nm are:

NA	n_{core}	n_{clad}	n_{core} / n_{clad}
0.11	1.4608	1.4567	1.003
0.22	1.4608	1.4441	1.012

Note: Stress can induce index changes, so the 0.11 NA fiber is more stress sensitive.

Our fiber lengths range from approximately 10 m at HEAF to 30 m at Site 300 to over 85 m at U1a. The transmission of the fibers, and the signal levels we obtain at the streak cameras, depends upon how long our fibers are:

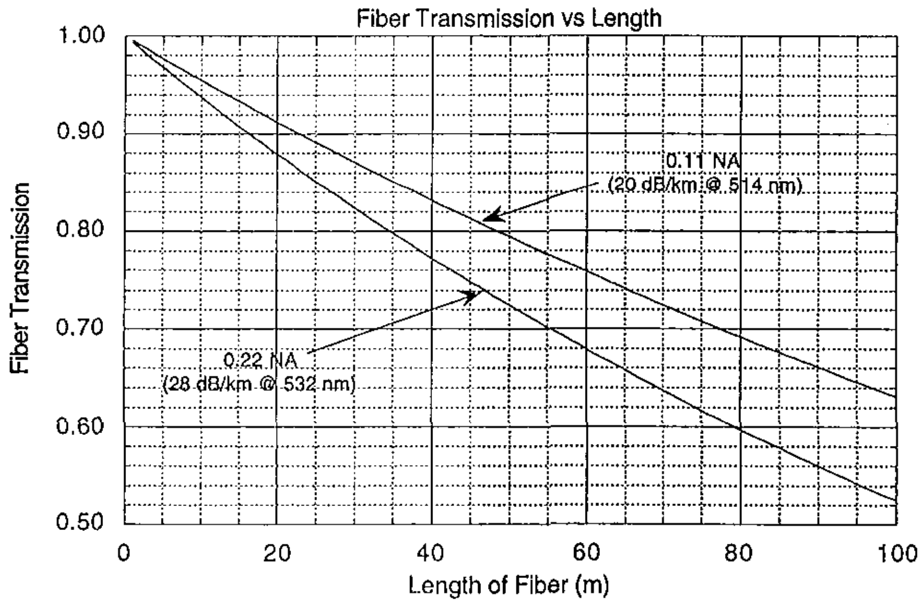
$$T = 10^{-\left(\frac{\alpha\left(\frac{dB}{m}\right)L(m)}{10}\right)} = e^{-\alpha(m^{-1})L(m)}$$

$$\alpha(m^{-1}) = 0.23 * \alpha(dB/m)$$

where T = transmission
α = attenuation coefficient
L = length of fiber

The labels on the sides of the spools of fiber give us the following attenuations:

0.11 NA	20 dB/km @ 514 nm
0.22 NA	28 dB/km @ 532 nm



Notes:

1) The 0.11 NA fiber is stress sensitive (Section 2.1), so the actual transmission may be less than given here. It is a good idea to always measure the transmission and NA of your fibers after they have been routed for your experiment.

2) Sometimes the ST connectors will be the cause of stress. If you are unsure, cut off the connectors and try again.

The number of modes N that a multimode fiber can carry depends upon the fiber core diameter d, fiber numerical aperture NA, and the wavelength of light λ. This formula for step-index fiber is an approximation for large numbers of modes:

$$N = \left[\frac{2\,d\,NA}{\lambda}\right]^2$$

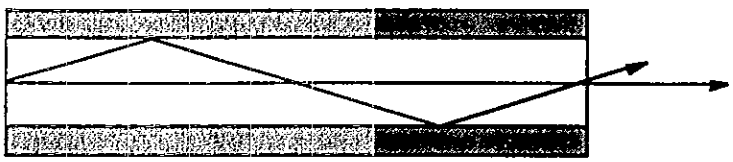
Some examples of mode numbers:

d (μm)	NA	λ (μm)	N
100	0.22	0.532	6840
100	0.11	0.532	1710
100	0.22	1.064	1710
100	0.11	1.064	428
50	0.22	0.532	1710
50	0.11	0.532	428

The speckle patterns that we observe when we shine the light from a fiber onto, say, a white card is the result of random interference among the different modes in the fiber. As the light exits the end of the fiber, the different modes become spatially overlapped on their way to the white card. Upon striking the card, constructive and destructive interference among the modes creates the pattern of bright and dark spots. This random interference inside the fiber is affected by many things, such as temperature changes, microcracks, wavelength changes, and stress. This is why we see the speckle pattern "boil" as we roll a fiber between our thumb and forefinger. The speckle contrast depends upon the laser linewidth and length of propagation in the fiber.* Narrow linewidth and short fibers produce high speckle contrast.

* CRC Fiber Optics, James C. Daly,1984, pp 93, 163.

Dispersion is the spreading of signals in fibers that results in a loss of time response. There are several types of dispersion. Material and chromatic dispersion are negligible for the Manybeam system. The most dominant form of dispersion in our system is modal dispersion. Modal dispersion is the time spread of a signal in fibers as a result of the different modes that exist in multimode fibers. Classically, in step-index fiber, this can be thought of as the time difference between a ray that travels straight through the fiber with no reflections and a ray that travels at the largest allowed angle and reflects many times.



Of course, no ray actually makes it all the way through a fiber without any bounces (unless that fiber were perfectly straight which ours are not), so a better way to describe modal dispersion is the time spread caused by the maximum and minimum group velocities (v_{max} and v_{min}) of the different modes*:

$$v_{max} = \frac{c}{n_{core}}$$

$$v_{min} = c \times \frac{n_{clad}}{n_{core}^2}$$

where n_{core} = index of refraction of the core
n_{clad} = index of refraction of the clad
c = speed of light in air/vacuum

Note that the expression for v_{max} is the speed of light in materials with index of refraction n_{core} given in section 8.1. This is actually the speed of light for the lowest order mode only. The expression for v_{min} is for the highest order mode allowed by the combination of core and clad index of refraction.

In step-index fibers with many modes, the resulting rms pulse spread is:

$$\frac{\tau}{L} = \frac{(NA)^2}{4nc}$$

where τ = time spread,
L = length of fiber,
NA = numerical aperture,
n = core index of refraction,
c = speed of light in air/vacuum.

For our 0.22 NA fiber (which carries the data information), τ/L = 28 ps/m.

* "Fundamentals of Photonics" by B.E.A. Saleh and M.C. Teich, John Wiley & Sons, Inc, 1991, p. 299.

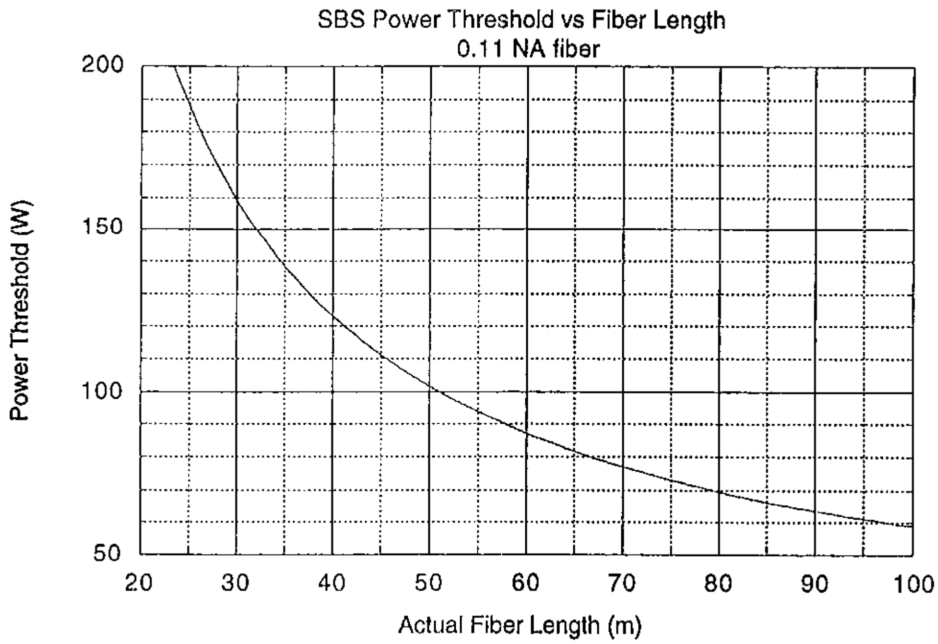
At high laser power levels, the light sets up a forward-traveling sound wave in the fiber. This sound wave creates density fluctuations in the core which reflects some of the incident light. Once past threshold, further increasing the incident power levels increases the transmitted light by only 10%, while the remainder of the excess light is reflected out the input end of the fiber. The SBS power threshold P_o is:

$$P_o = \frac{21\pi d^2}{4 g_B L_{eff}}$$

where $d = 100\ \mu m = 1e\text{-}4\ m$ = the fiber core diameter
$g_B = 3.8e\text{-}11\ m/W$ = Brillouin gain which depends upon material
L_{eff} = effective fiber length given by:

$$L_{eff} = \frac{1-e^{-\alpha L}}{\alpha}$$

where $\alpha = 28\ dB/km = 0.0064\ m^{-1}$ = absorption coefficient for 0.11 NA fiber
L = actual fiber length

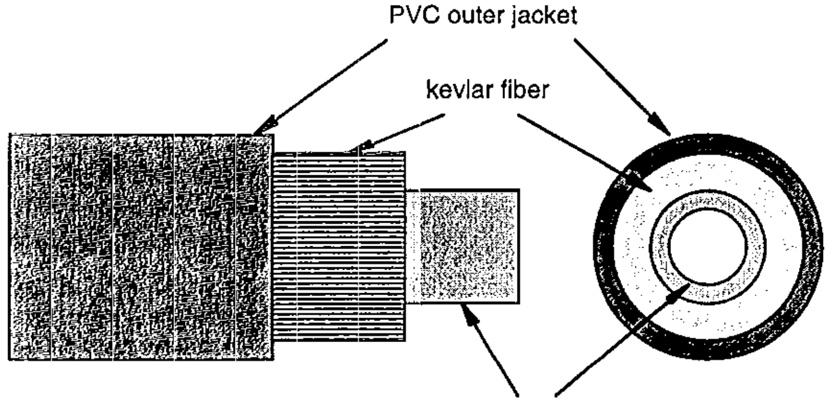


Note: This discussion is from the memo "Fiber Power Limitation via Stimulated Brillouin Scattering" by Leland Collins and Cheng-Huei Lin, 10/29/97.

When assembled with probes and ready for a shot, our fibers are protected from damage by heavy-duty jackets called furcation tubes. Our furcation tubes are made by:

Northern Lights Cable, Inc.
North Bennington, VT 05257
ph# 802-442-5411

These tubes have three layers--an outer jacket made of PVC, a layer of kevlar yarn for pull strength, and an inner tube made of polypropylene. We also order the optional pull string to make it easy to pull the fibers into the furcation tubes. As a convention for easy identification, we order yellow outer jackets for the 0.11 NA fibers and orange outer jackets for the 0.22 NA fibers. Northern Lights will also enclose the yellow and orange jackets in a single outer jacket which they call bifurcated tubing. We recently started requesting this because it is much easier to handle.



The approximate diameters (mm) are:

	Inside Diameter	Outside Diameter
Outer jacket	2.5	3.0
Inner tube	1.04	1.8

Section 3

Probes

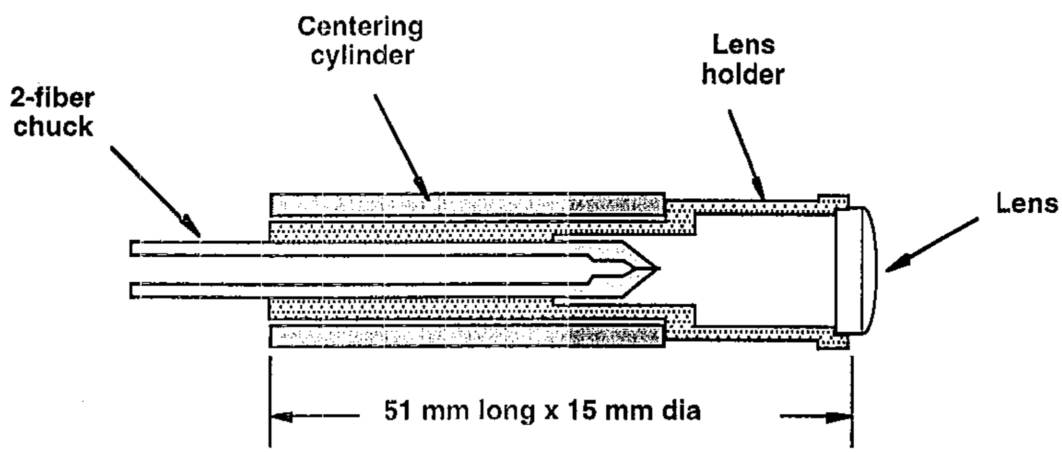
The probes are a set of brass parts that hold the fibers and lenses aligned with respect to the aiming points on the experiments. These probes use two different types of fiber and custom-designed lenses. This combination of fibers and lenses provides higher efficiency than other types of probes.

The probes are designed to be held a fairly large distance away from the experiment; our standard distance is 250 mm.There are advantages and disadvantages to this concept:

Advantages:
- large depth of field so that a surface can be followed for 10's of mm
- leaves room near the experiment for other diagnostics, especially imaging
- small spot size on the surface (< 1 mm dia)

Disadvantages:
- low probe efficiency--on the order of 10^{-4}
  (eg, 50 W in yields 5 mW out, which is why we need a big laser)
- probe alignment and focus can be time-consuming
- custom lenses are difficult to obtain

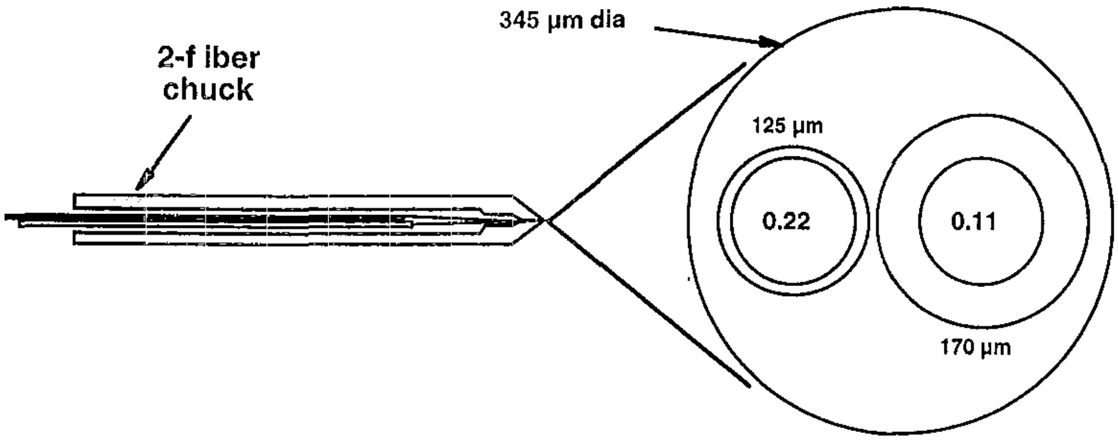
The probes have four main pieces:



Note: The dimensions may be obtained from the following drawings:

Part	Drawing #
2-fiber chuck	98-101647
Centering cylinder	98-101649
Lens holder	98-101646

The 2-fiber chuck is approximately 2.5" long x 0.25" diameter. One 0.11 NA launch fiber and one 0.22 NA receiver fiber are epoxied into the chuck. The ends of the fibers are held in place by a small hole in the tip approximately 345 μm diameter.

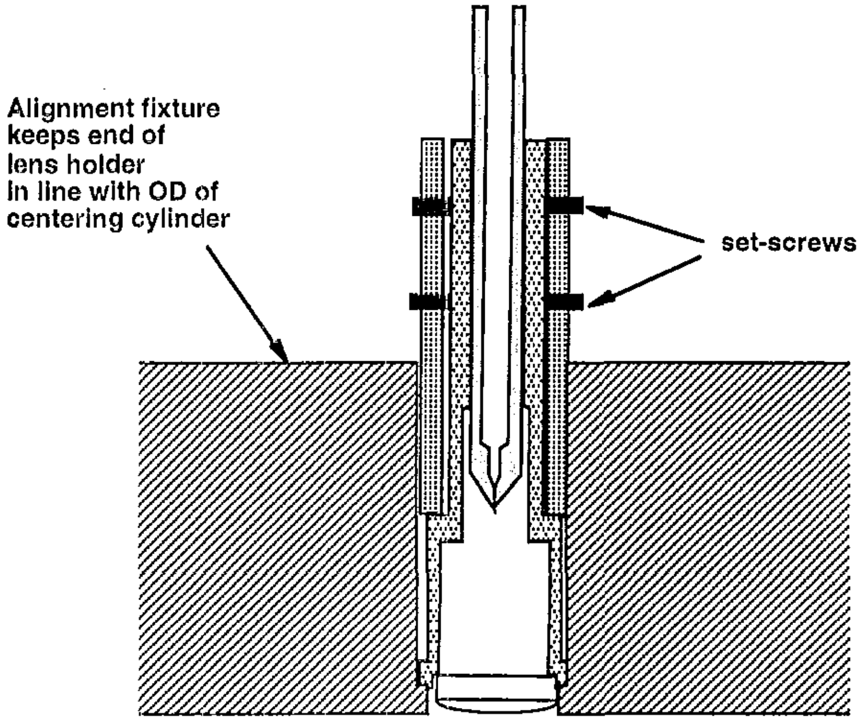


Notes:

1. Neither fiber core can be centered on-axis with this geometry.

2. The spacing between the fiber cores may vary somewhat because the hole is oversized. The nested lens is designed for a core spacing of 147 μm, so the probe efficiency may be less than optimum if the fiber tips are not touching each other.

The centering cylinder is approximately 1.40" long x 0.6" diameter. There are two sets of 4 set-screws tapped around the cylinder to allow for tilt of the lens holder. These are adjusted to eliminate "run-out", that is, the lens holder is tilted until the light from the 0.11 NA fiber is focused on-axis with the centering cylinder.



Notes:

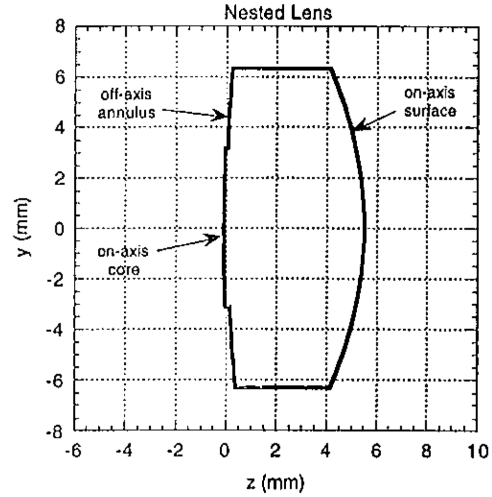
1) The OD of the centering cylinder and the ID of the hole in the mounting bracket on the shot must be specified to high precision to assure that the probe is aimed within 0.5 mm of the desired location:

- OD of the centering cylinder: 0.5880 ± 0.0005 inches.
- ID of the hole in the bracket: 0.5914 ± 0.0003 inches.

2) When setting the runout, care must be taken to avoid bending the probe. It is possible to overconstrain the set-screws so that the probe will not fit into the bracket hole. Suggestion: back out the lower set of screws and adjust the runout with only the upper set. Then, carefully tighten the lower set as you watch the aim and make sure the aiming point does not move.

The nested lenses were custom-designed by David Goosman and he has applied for a patent on the design. The lenses have an offset annular region so that both fibers placed side-by-side in the two-fiber chuck "see" the same spot on the experiment surface. These lenses are made of acrylic with index of refraction = 1.4946. They are made by Nylomold Corporation in Rochester, NY (www.nylomold.com).

A small amount of light launched toward an experiment is reflected or scattered from the nested lens and enters the 0.22 NA return fiber. This is a source of undoppler-shifted light on our streak records. Normally, this amounts to approximately 2 - 12 ppm, depending upon speckle locations, but drops to 1 ppm when the lens is AR coated (Section 8.8).



Notes:

1) The offset of the annulus is set for using the probe at a distance of 250 mm. At this distance, the probe has a magnification of approximately 10. That is, the 0.1 μm diameter fiber core images to a 1.0 mm diameter spot on the experiment surface.

2) When the probe is used at distances other than 250 mm, the spot size changes linearly with the distance, e.g., at 200 mm, the spot size is approximately 0.8 mm diameter.

3) When built into a probe and adjusted, these lenses provide a distance at which the efficiency of the probe is maximized and a different (longer) distance at which the spot diameter is minimized. The difference in these distances is called "skew" (Section 3.5).

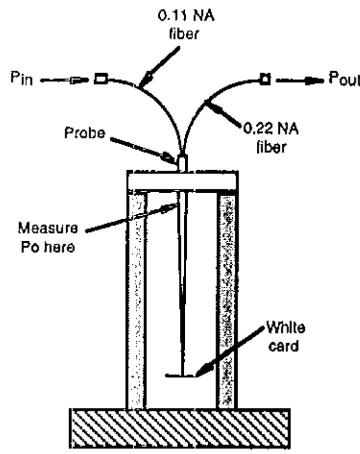
4) There are 3 different versions of the nested lens in use--they are designated by the letters D, J, and G--and each lens provides a different skew. The value of skew is always referred to the case when the probe is set for maximum efficiency at 250 mm.

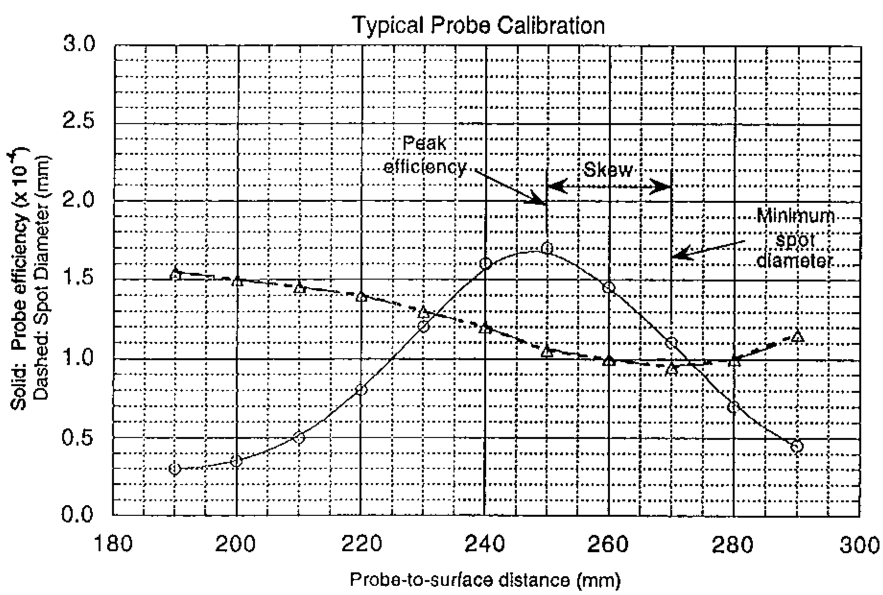
Lens Type	Skew (mm)
D	40
J	15
G	20

The probe calibration values are standardized to a peak-efficiency distance of 250 mm on a white card, and includes the effect of four Fresnel reflections as the light enters and leaves the probe:

$$E = \frac{1}{0.865}\frac{P_{out}}{P_o}\left(\frac{Dist(mm)}{250}\right)^2$$

where Dist(mm) is the distance from the end of the lens holder to the white card. A standardized efficiency value for a good probe is approximately 1.5 x 10^{-4}.





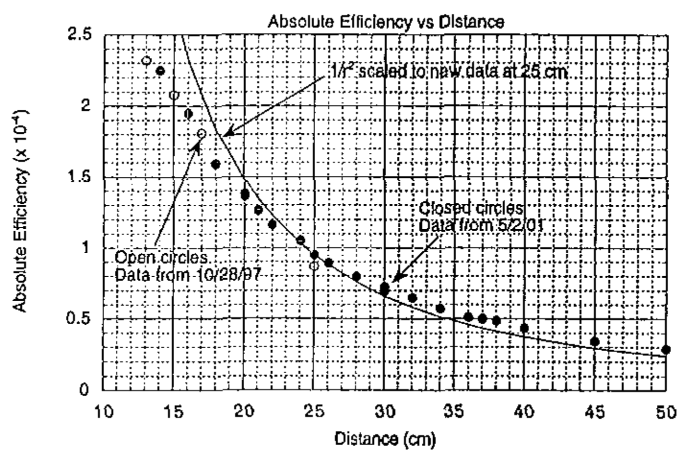
Note: If the light budget is being calculated, then the standardized efficiency given above is not the correct quantity to use. Instead, use the absolute efficiency given by:

$$E_{abs} = \frac{P_{out}}{P_{in}}$$

Our probes were designed for a peak efficiency distance of 25 cm. This is determined by the amount of offset in the annular part of the nested lens with respect to the core. (Section 3.4). The absolute efficiency (defined at the bottom of Section 3.5) of a probe set for 25 cm is approximately 1×10^{-4}. There are some applications in which a higher efficiency is desired, such as obtaining velocities from surfaces with very low reflectivity. In these cases, the easiest solution might be to move the probe closer to the experiment. In other cases, the probes might need to be moved further away to allow an unobstructed view by an imaging diagnostic.

The following graph shows peak efficiency data taken three and a half years apart with two different probes looking at a white card. This data was taken by setting the white card at a given distance from the probe and adjusting the probe to find the highest efficiency possible. The white card was then moved to a new distance from the probe and the new peak efficiency was found. The data closely follow a $1/r^2$ dependence at distances greater than 20 cm, but fall below the $1/r^2$ dependence at shorter distances.

The graph shows absolute efficiency versus distance, which is the quantity needed to calculate signal levels.



The disadvantage of moving the probe closer to the experiment is that the surface cannot be followed for as long a distance. The FWHM of the probe efficiency decreases as the peak efficiency distance decreases. Some examples:

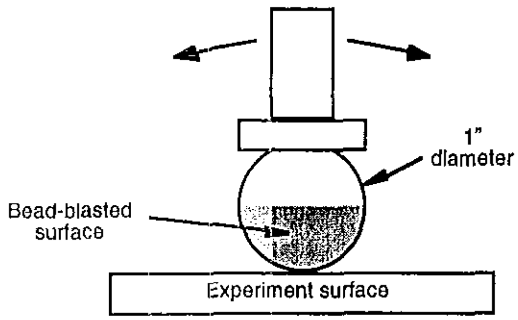
Peak Eff Dist (cm)	FWHM (cm)
17	2.8
25	5.5
36	11.0

This is a trade-off which must be decided on a case-by-case basis.

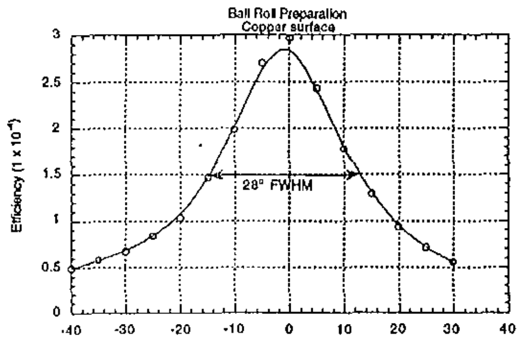
It is important that the fringe brightness remains nearly constant during the course of a measurement. Unprepared surfaces, such as nearly specular finishes or machine-grooves, often show a sudden decrease in fringe brightness when the initial shock arrives at the probe aiming point. Even roughing the surface with sandpaper does not always provide constant fringe brightness at shock arrival.

We have developed a method of preparing the surface at the probe aiming points which provides nearly constant fringe brightness during an experiment. This technique, called ball rolling, produces a surface finish with a dull mat appearance. A hard stainless steel ball (tooling ball, actually) is thoroughly bead blasted until no specular reflections can be seen on its surface. This ball is then rolled back and forth on the surface of the experiment until a uniform mat finish is produced. Usually a mask is made to locate where the ball rolling should be done on the surface and to keep the ball rolling inside a well-defined area around each aiming point.

Important!
1) Do not deform the surface. Use only just enough pressure to produce the mat finish.
2) Do not let the ball slip on the surface. This will produce a relatively deep groove.



When the probe shines on the ball-rolled surface, the light scatters from the surface with an angular distribution that has approximately 30 degrees FWHM.



Notes:

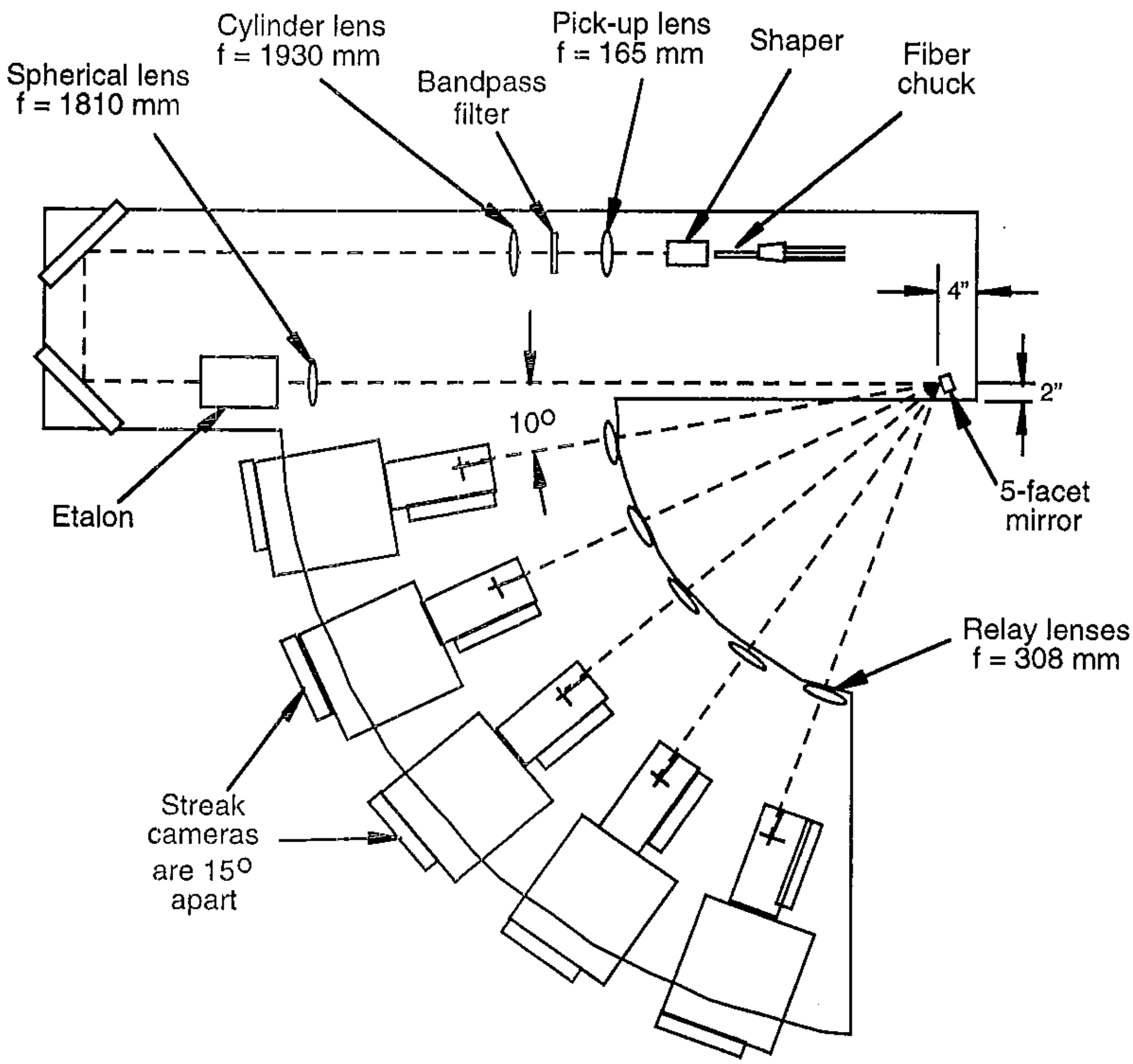
1) We usually set up our experiments so that the probes are within 10 degrees of the surface normal at the start of the measurement.

2) The probes are usually set so that the surface normal tilts toward the probe during the course of the experiment.

Section 4

Analyzer Table

The analyzer table contains the optics that convert the wavelength changes of the doppler-shifted light into angle changes that are recorded on the streak cameras. (The analyzer table is also called the tau table.) The basic layout of the analyzer table and the focal lengths of the important lenses are shown below:



Fiber chuck to 5-facet mirror imaging:

Magnification = 2.742
Fiber spacing = 0.0134" = 339.5 μm
Image spacing at facets = 0.0367" = 931 μm

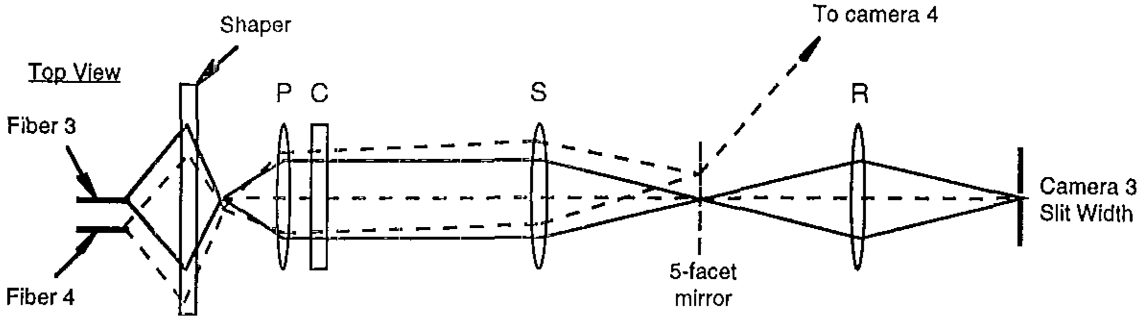
Relay lens distances: 5-facet mirror to slit = 55.00" = 139.7 cm
5-facet mirror to relay lens = 36.67" = 93.1 cm
Relay lens to streak camera slit = 18.33" = 46.6 cm
5-facet mirror to streak camera slit mag = 0.50 approximately

Flight time for speed of light air path = 32 ns.

The optics on the analyzer table may be confusing at first, so a simplified look at the imaging from the fiber array to the streak cameras may be useful. To do this, let's remove the large turning mirrors so that all the optics are in a straight line and remove the etalon so that we can see where the light goes without worrying about forming fringes. Because we have cylindrical optics on the analyzer table, the top view looks different from the side view. We'll look at the top view first.

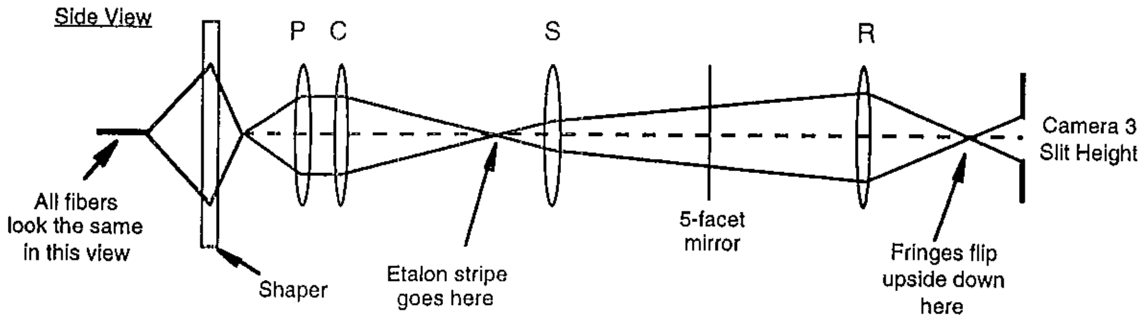
Note: In the following sketches, P = spherical pickup lens, C = cylinder lens, S = spherical lens after the etalon, and R = relay lens. The sketches are not to scale.

Top view: In a nutshell, each fiber is imaged to its own facet at the 5-facet mirror. The 5-facet mirror then reflects the light from each fiber to its own streak camera slit.



From the top view, we can see the row of fibers in the fiber chuck (section 4.3). The light from the fibers enters the shaper. The shaper is not a simple optic and is discussed in section 4.5. For now, all we need to know is that the shaper is a 4:1 reducer (section 8.4) when looking from the top. This makes the images of the fibers 25 μm wide instead of 100 μm. It also makes the distance between the images one-fourth that of the fibers themselves. (This second effect is very important in reducing the range of horizontal angles propagated through the remainder of the analyzer table. This minimizes walkoff in the etalons (section 4.6.1) and minimizes finesse degradation at the streak camera slits (section 4.6.5).) The output images from the shaper are in the focal plane of the pickup lens, so we have parallel light leaving the pickup lens (section 8.3). From the top view, the cylinder lens has no power, so the parallel light continues to the spherical lens (no etalon for the moment). Because the spherical lens receives parallel light, this lens focuses the light at its focal plane (section 8.3), which is where we put the 5-facet mirror. The width of the images at the 5-facet mirror (approximately 274 μm) is given by multiplying the fiber core diameter times the ratio of the focal length of the spherical lens to the focal length of the pickup lens (section 8.4) and then dividing by four because of the shaper. Each fiber produces its own image at the 5-facet mirror, so we have five narrow images at the 5-facet mirror. Each facet of the 5-facet mirror reflects its own image to a streak camera. The relay lens focuses the image from the 5-facet mirror to another image at the streak camera slit with a 2:1 demagnification (sections 8.3 and 8.4), so that the images are approximately 137 μm wide at the streak camera slits.

Side view: In a nutshell, each fiber is imaged to the stripe at the etalon. The light is then imaged in front of the streak camera slit, so that the light fills the height of the slit.



From the side, we see only the edge of the first fiber, so the light from all the fibers appears to do the same thing in this view. The fibers send light into the shaper. The shaper is not a simple optic and is discussed in section 4.5. From the side view, the shaper has a 1:1 magnification (section 8.4), so the images of the fibers here are the same height as the fiber cores themselves. The output images are in the focal plane of the pickup lens (just like in the top view), so we again have parallel light leaving the pickup lens (section 8.3). From the side view, the cylinder lens has power, so it takes the parallel light from the pickup lens and produces an image at its focal plane (section 8.3), which is where we put the stripe of the etalon (but not just yet). The height of the images at the stripe (approximately 1.17 mm) is given by multiplying the diameter of the fiber cores times the ratio of the focal length of the cylinder to the focal length of the pickup lens (section 8.4). With no etalon in the way, the rays continue (diverging now) to the spherical lens. The image at the etalon stripe is only 350 mm or so from the spherical lens, whereas the focal length of the spherical lens is 1810 mm. The spherical lens does not focus these rays but only decreases the divergence of the rays on their way to the 5-facet mirror (28-mm-high bundle of rays here) and then on to the relay lens (40-mm-high bundle of rays here). The relay lens focuses these rays to a point approximately 125 mm in front of the streak camera slit (section 8.3), so that the rays cross over and diverge on their way to the slit. This produces a line of light approximately 15 mm high at the slit (from section 5.2.1, recall that the slit is 11 mm high).

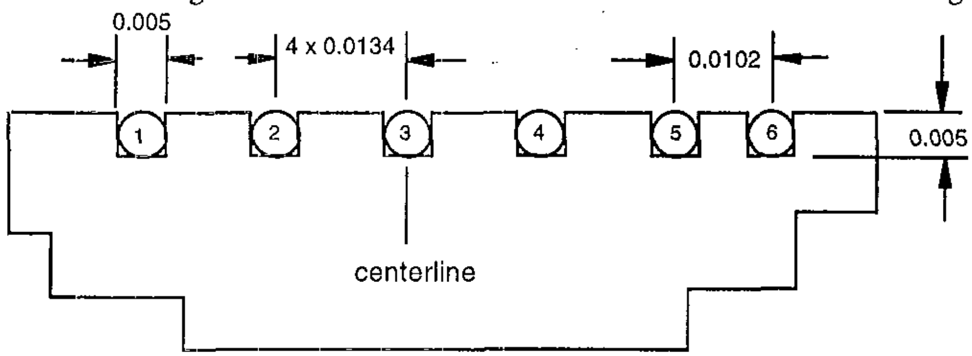
Etalon: Now let's put the etalon back in. From section 4.6.3, we know that only parallel light leaves the etalon at discrete angles. This parallel light arrives at the spherical lens which focuses the light to an image at its focal plane (section 8.3), which is where we put the 5-facet mirror. Recall that this is the same place the light was focused by the other optics when looking at the top view. The discrete angles of parallel light leaving the etalon are focused to a series of concentric rings at the 5-facet mirror.

Summary: We have a situation where the optics without the etalon want to produce a series of five vertical columns of light at the 5-facet mirror. The etalon, however, wants to produce only concentric rings at the 5-facet mirror. The result is that light is actually produced only at those places that are allowed by both the optics and the etalon. We end up with 5 vertical columns of fringes at the 5-facet mirror. The mirror facets then send the fringes on to each streak camera slit.

The fiber chucks hold the fibers in a linear array. The locations of these fibers define the input of the light to all the optics on the analyzer table, therefore, the precision of the fiber locations is important. The fibers need to be spaced properly to map correctly onto the 5-facet mirror, and the fibers need to be in a linear array to map properly onto the stripe of the etalon. There are several chuck designs in use on different analyzer tables.

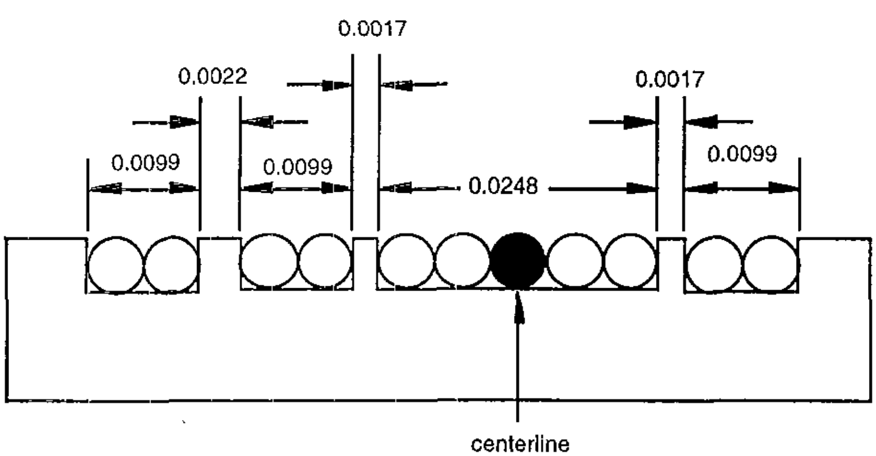
6-fiber chuck

This is the original chuck design with one fiber per camera plus an alignment fiber. The table is aligned with fiber #3 defining the optical axis. The sixth fiber is used to check the table alignment with the 5-hole card after the shot fibers are hooked up. The entire chuck is pulled over a precise amount so that the #6 fiber ends up in the #3 position to perform that check. Sometimes, the sixth fiber is used to duplex the data from two probes onto camera 5. There is a 10-fiber version of this chuck design that has half the fiber spacing and no additional fiber for alignment checks. The dimensions are in inches in the following sketch.



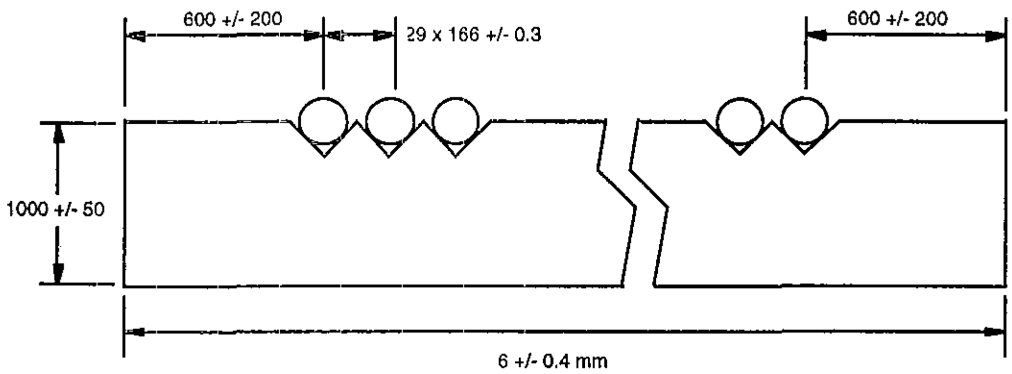
11-fiber chuck

This chuck is designed to duplex all five cameras on a table and still have a fiber to define the optical axis during table alignments. The black fiber location is used for table setup and never needs to be disconnected, so it is always easy to check alignment with the 5-hole card even when the shot fibers are hooked up. The disadvantage of this design is that the fibers do not map onto the centers of each facet of the 5-facet mirror, so that aligning the mirror is critical. The dimensions are in inches in the sketch.



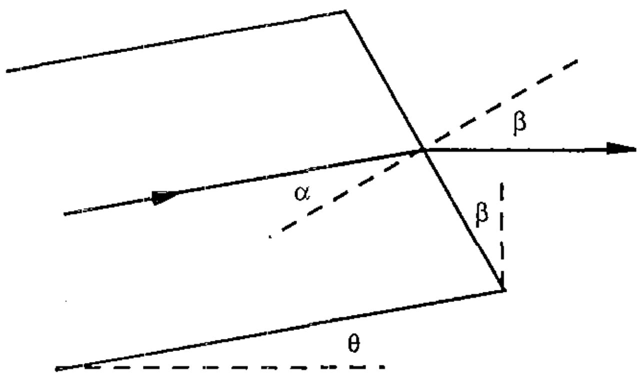
Silicon V-grooves

All of the fiber chucks to date have been made of brass with the grooves machined as precisely as possible. In all chucks, however, it is possible to see that the fibers are not positioned as precisely as desired. The newest idea is to use silicon etching technology to provide very precise placement of the fibers in a linear array. The first silicon v-groove chip was designed by Rex Avara and Anthony Rivera. Each chip is 10 mm long x 6 mm wide and has 30 grooves with a spacing that allows all five cameras on a table to be duplexed at once. The large number of total grooves was chosen to allow for flexibility in any future chuck designs. The dimensions in the following sketch are in microns except where noted.



Some of the light from the fibers unavoidably reflects off various pieces of optics and returns to the fiber chuck. If the ends of the fibers, and the end of the fiber chuck, were polished normal to the chuck axis, then much of this light would reflect off the brass housing and be returned to the table. This doubly-reflected light could end up on streak cameras which are not the ones for which it was intended. This would be a source of crosstalk among the data channels.

To minimize this effect, the fiber chucks are tilted upward at an angle of 10 degrees. The fiber endface must then be polished at an angle to make the light exit the fiber with the central ray horizontal. The following sketch shows the geometry:



Using Snell's law (section 8.2), it is easy to calculate the angle β knowing that $\theta = 10°$ and the index of refraction of the fiber core n = 1.46:

$$\tan(\beta) = \frac{n*\sin(\theta)}{n*\cos(\theta)-1}$$

$$\tan(\beta) = 0.579$$
$$\beta = 30.1°$$

β is the angle with respect to the vertical and we want the angle with respect to the fiber itself, which is $\beta - \theta = 30.1° - 10° = 20.1°$.

We polish the tips of our fiber chucks at an angle of approximately 20°.

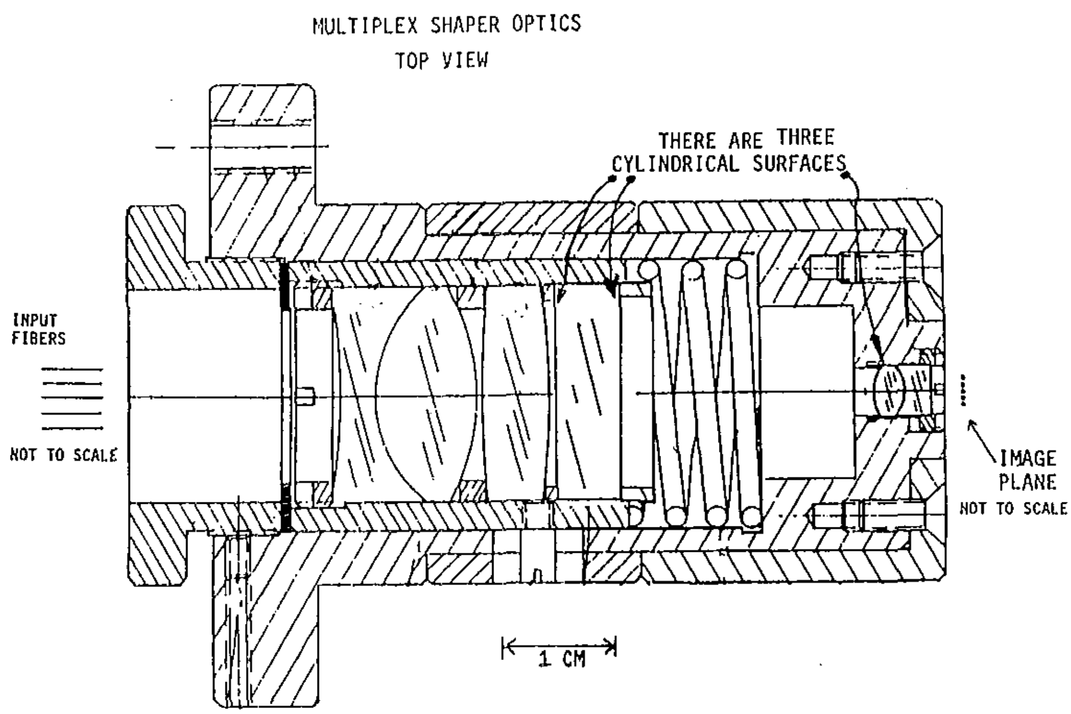
The shaper is the first set of optics after the fiber chuck. Its purpose is to provide 1:1 imaging in the vertical direction and 4:1 imaging in the horizontal direction. In the image plane of the shaper, then, the spacing between the fibers is reduced by a factor of 4, and the circular endfaces of the fibers become ellipses with vertical-to-horizontal aspect ratio of 4:1. These elliptical images are called "footballs". The shaper accepts the 0.22 NA light from the fibers and maintains that NA both vertically and horizontally beyond its image plane.

The advantages of using the shaper are:

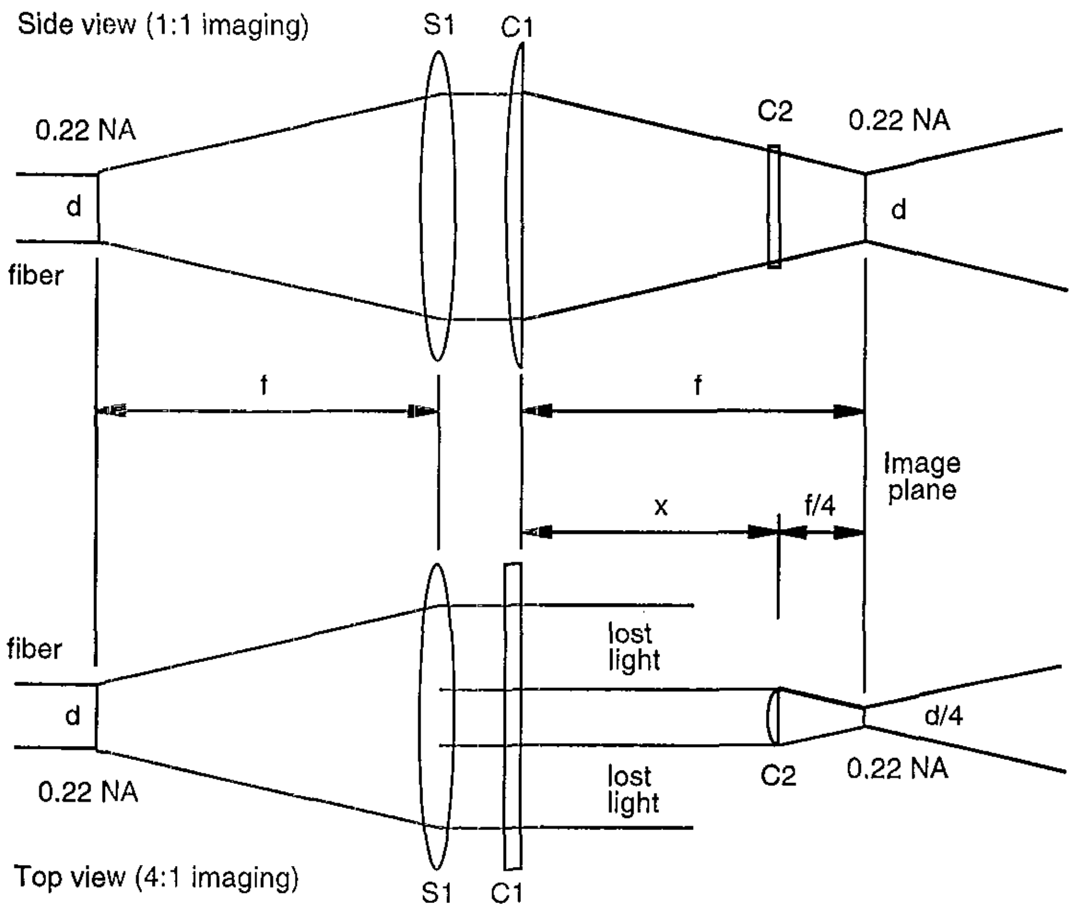
- more light into the streak camera slits,
- narrower fringes at the 5-facet mirror to allow duplexing,
- smaller range of horizontal angles in the etalon for less walkoff.

The disadvantages of using the shaper are:

- quite expensive,
- somewhat difficult to align properly,
- unavoidable light loss with transmission of 38%.



The shaper actually contains six lenses. Some of the lenses are for aberration corrections, so functionally, the shaper can be represented as a set of only three lenses--a spherical lens and two crossed cylinder lenses.



Notes:

1) S1 is a spherical lens, C1 and C2 are cylindrical lenses

2) The focal length of S1 is the same as for C1 for the 1:1 imaging.

3) The focal length of C2 is one-fourth as for C1 for the 4:1 imaging.

4) The dimension x must be adjusted so that the 4:1 image plane is coincident with the 1:1 image plane.

5) The axes of C1 and C2 must be perpendicular to each other.

6) The axis of C1 must be horizontal.

Section 4.6

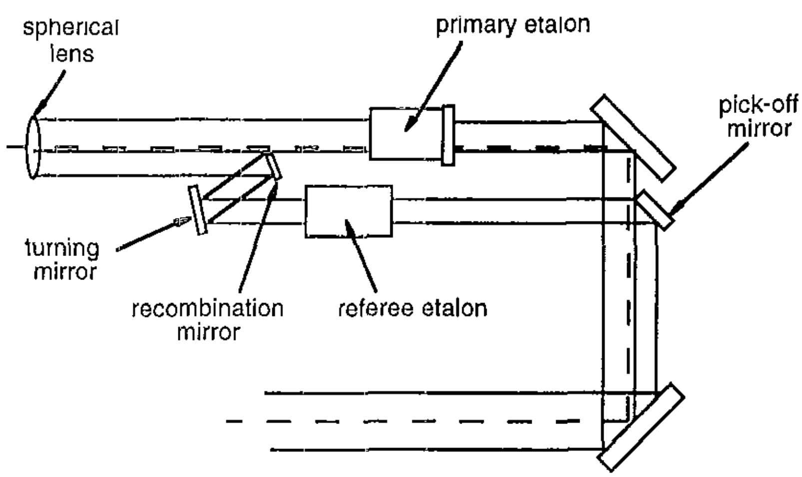
Analyzer Table
Etalons

Etalons are made from two mirrors that are very flat and very parallel to each other. For most of our etalons, the mirrors have very high reflectivity. For example , the front mirror may have reflectivity $R_1 = 99\%$, and the back mirror may have $R_2 = 92\%$. If light enters the etalon at low angles, it will reflect back and forth many times. If light enters the etalon at high angles, the light will walk out from in between the mirrors in only a few bounces. On the analyzer table, the geometry of the shaper, pickup lens, and cylinder lens was chosen to use a small (± 6 mrad) range of angles vertically and nearly parallel light horizontally. This means that the light should have very little "walkoff", that is, the light will undergo many bounces before being lost. This greatly improves the finesse of the fringes formed by the etalon.

At the different facilities with Manybeam systems, we have various types of etalons. Some etalons have only air between the two mirrors (air cavity), while other etalons are blocks of solid glass with mirror coatings on two opposite surfaces (glass cavity). Other etalons are air cavities with a small piece of very high-quality glass inserted in between the mirrors to provide a built-in referee cavity. Some of the air cavities are fixed etalons and others are tunable.

The fixed air cavities are made by TecOptics and the tunable air cavities are made by Burleigh. The glass cavities are made by TecOptics and Queensgate.

Nearly all of the analyzer tables operate with a primary and referee cavity to determine velocities after jump-off (see section 7.1.3). The air cavities with glass inserts are primary and referee cavities built into a single unit. With air-only and glass-only cavities, we build a "roundabout" system that separates the light into two beams, one for the primary cavity and one for the referee cavity, and then recombines the beams again. The following sketch shows how a roundabout might be set up:



Notes:

1) We generally ask that the mirrors be flat to $\lambda/100$ and parallel to $\lambda/50$. This means the light will undergo tens of bounces inside the etalon at low angles and assures a finesse (Section 4.6.5) of at least 20.

2) Many of our etalons are 50 mm to 60 mm long. The wavelength of our light is 532 nm, so these etalons are approximately 100,000 wavelengths long.

When light from our probes reflects or scatters from a moving surface, the frequency of the light is shifted according to how fast the surface is moving. Usually, the surfaces we measure are moving toward the probes, so the frequency is usually shifted to a higher frequency. This change in frequency Δf is called the Doppler shift, and is given by:

$$\Delta f = f_1 - f_0 = 2\frac{v}{c} f_0$$

where f_0 = original frequency from our doubled YAG laser
f_1 = Doppler-shifted frequency from the moving surface
v = velocity of the moving surface
c = speed of light

For our experiments, we can use $c=f\lambda$ to calculate the frequency of the light from our laser. We know (section 9.1) that the wavelength of the doubled YAG is 532.1 nm and the speed of light is 299.8 mm/ns, so we have:

$$f_0 = \frac{c}{\lambda} = \frac{2.998x10^8 m/s}{532.1x10^{-9} m} = 5.63x10^{14} Hz$$

For a surface velocity v = 1 mm/μs, Δf = 3.76 x 10^9 Hz or 3.76 GHz.

If we look at the Doppler shift in terms of wavelength, then the change in wavelength Δλ is:

$$\Delta\lambda = 2\frac{v}{c}\lambda_0$$

where λ_0 = 532.1 nm from our laser and the new wavelength is shorter when the surface moves toward the probe. For a surface velocity v = 1 mm/μs, Δλ = 0.0036 nm.

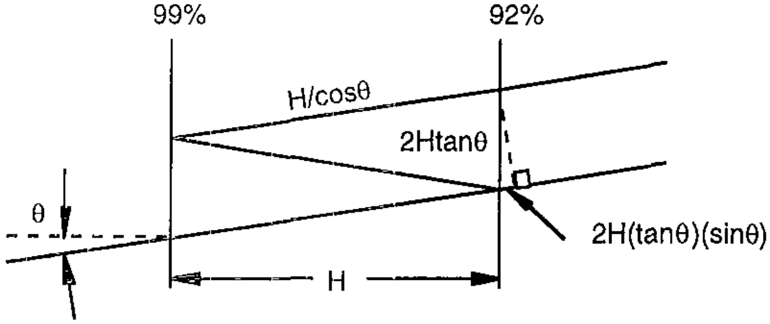
Notes:

1) Many of our experiments involve velocities on the order of 1 mm/μs. For v = 1 mm/μs, the quantity 2v/c = 6.7 x 10^{-6}.

2) For typical velocities on the order of 1 mm/μs, the Doppler shift is only 1/150,000 of a wave and our etalons are on the order of 100,000 waves long (from Section 4.6.1). Our etalons must be very high quality to measure such a small wavelength change.

3) The Doppler shift we measure is actually determined by the component of velocity in the direction of the probe. If there is a fairly large angle between the direction of the light going to the probe and the direction that the surface is moving, or if this angle changes significantly during the measurement, then the final analysis should take this into account.

Constructive interference among all the bounces inside the etalon means that only parallel light leaves the etalon. Furthermore, that light is emitted at only discrete angles--this is what forms our fringes.



The light leaving the etalon from consecutive bounces must be in phase; that is, the path length difference between adjacent bounces must be an integer number of wavelengths. For an air cavity:

$$m\lambda = 2H/\cos\theta - 2H \tan\theta \sin\theta$$
$$m\lambda = 2H \cos\theta$$

Notes:

1) As the surface moves toward the probe in an experiment, λ decreases due to the Doppler shift described in Section 4.6.2. When λ decreases, then $\cos\theta$ decreases, and θ increases. On our streak camera records, the fringes move away from the horizontal center as the velocity increases.

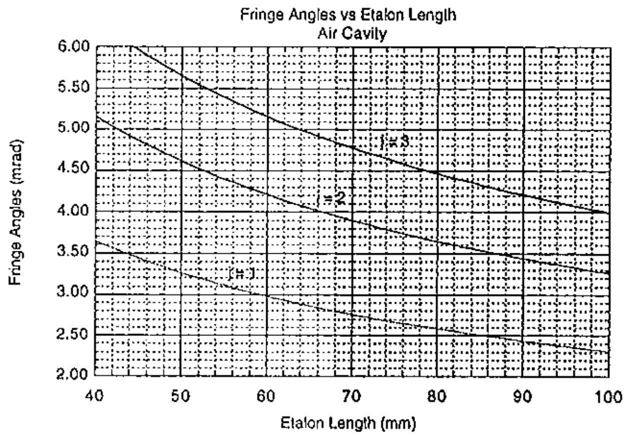
2) The fringe angles θ_j emitted from an etalon are given by:

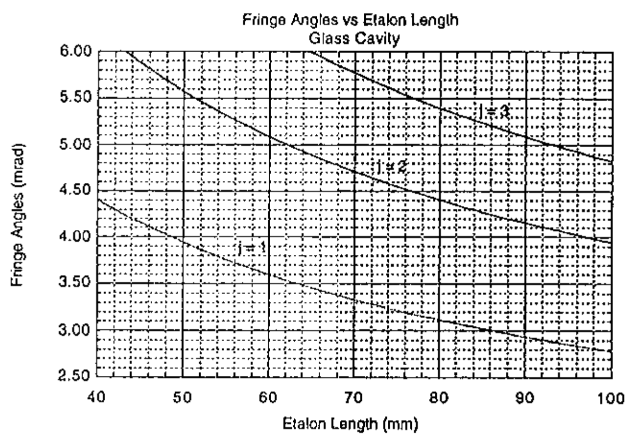
$$\theta_j = \sqrt{\frac{j\lambda}{H + \frac{T}{n}}}$$

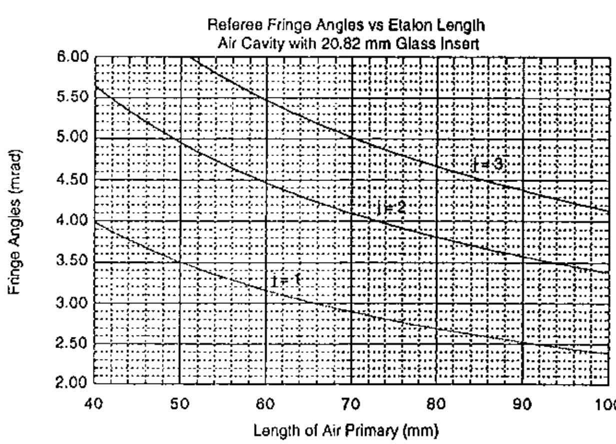
where

- j = fringe order
- λ = wavelength
- H = length of air gap in the etalon
- T = length of glass in the etalon
- n = index of refraction of the glass = 1.46071

Fringe orders: The following plots give the first three integer fringe orders allowed by etalons as a function of etalon length. The top graph is for all-air cavities, the middle graph is for all-glass cavities, and the bottom graph is for air cavities with a glass insert of thickness 20.82 mm.







Maximum fringe orders: The height of the streak camera slits is 11 mm, which maps to a column of fringes 22 mm high at the 5-facet mirror. The fringes that can enter the streak camera slit and be recorded as data, then, must be within 11 mm of the centerline of the 5-facet mirror. Recall from section 4.1 that the focal length of the spherical lens after the etalon has a focal length of 1810 mm. Section 8.5 shows that the maximum angle θ_{max} from the etalon that can be recorded as data is:

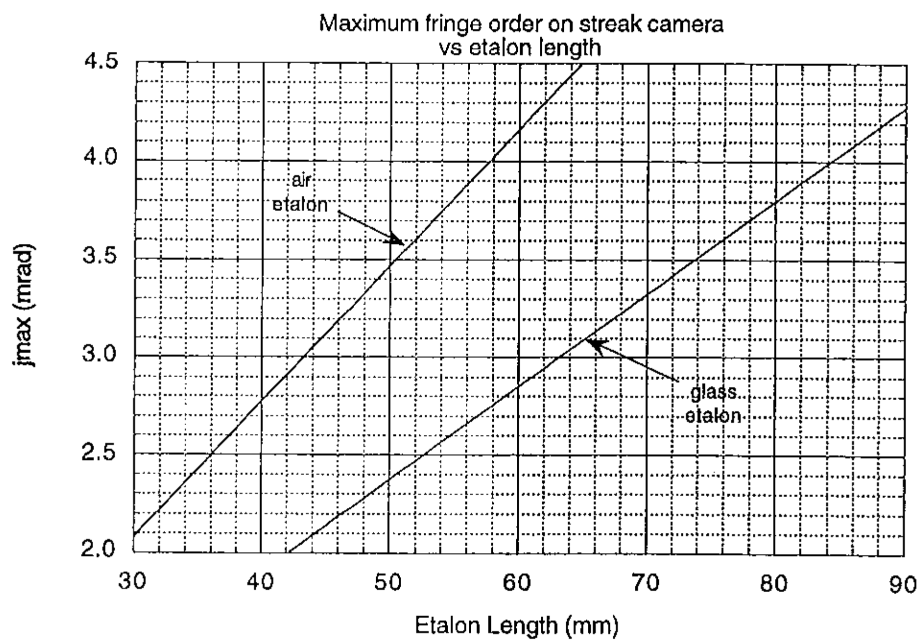
$$\theta_{max} = \frac{11}{1810} = 6.08 mrad$$

Substituting θ_{max} for θ and j_{max} for j in the equation at the bottom of the first page of this section, and solving for j_{max}, allows a calculation of the maximum fringe order recorded as data for etalons of different lengths.

For an air cavity of length H: $$j_{max} = \frac{\theta^2_{max}}{\lambda} H = 0.0694 * H(mm)$$

For a glass cavity of length T: $$j_{max} = \frac{\theta^2_{max}}{n\lambda} T = 0.0473 * T(mm)$$

where n = index of refraction of the etalon glass = 1.46071, and λ = 0.0005321 mm.



The different angles at which light is emitted from an etalon are called fringe orders and are labeled with the letter j. The difference between two fringe orders is called the fringe constant and is generally a few milliradians. Knowing the length of the etalon and the wavelength of light, we can convert the fringe constant f_c to a velocity with units of mm/μs:

$$f_c = \frac{c\lambda}{4\left[H + T\left(n - \lambda \frac{dn}{d\lambda}\right)\right]}.$$

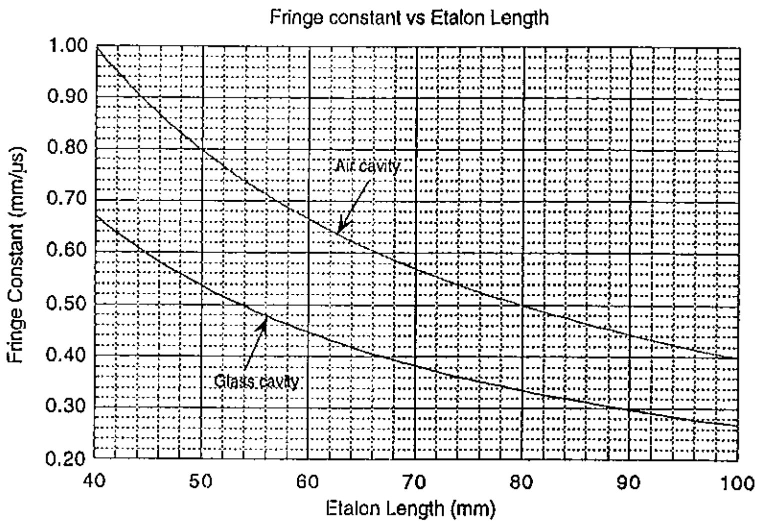
$$f_c[mm/\mu s] = \frac{39.88}{H[mm] + 1.485 * T[mm]}$$

where c = speed of light in air/vacuum
λ = wavelength of light from the laser = 532.1 nm
H = length of air in the cavity
T = length of glass in the cavity
n = index of refraction of the glass = 1.46071
dispersion at 532 nm is

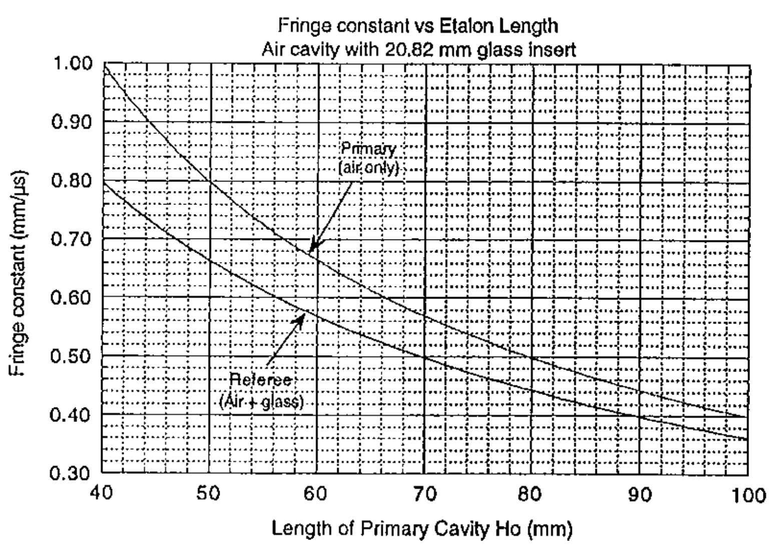
$$\lambda \frac{dn}{d\lambda} = -0.024$$

Note: The fringe constant is used in the calculation of velocity from the data records (see section 7.1.1).

In the following graph, the etalons are all-air or all-glass:



In the following graph, the primary cavity is all-air with length Ho, and the referee cavity is air+glass with glass insert T = 20.82 mm and air length H = (Ho - 20.82) mm:



Finesse is a measure of how good an etalon is. An etalon with very flat and parallel mirrors will produce very narrow fringes. The higher the finesse number, the higher quality the etalon.

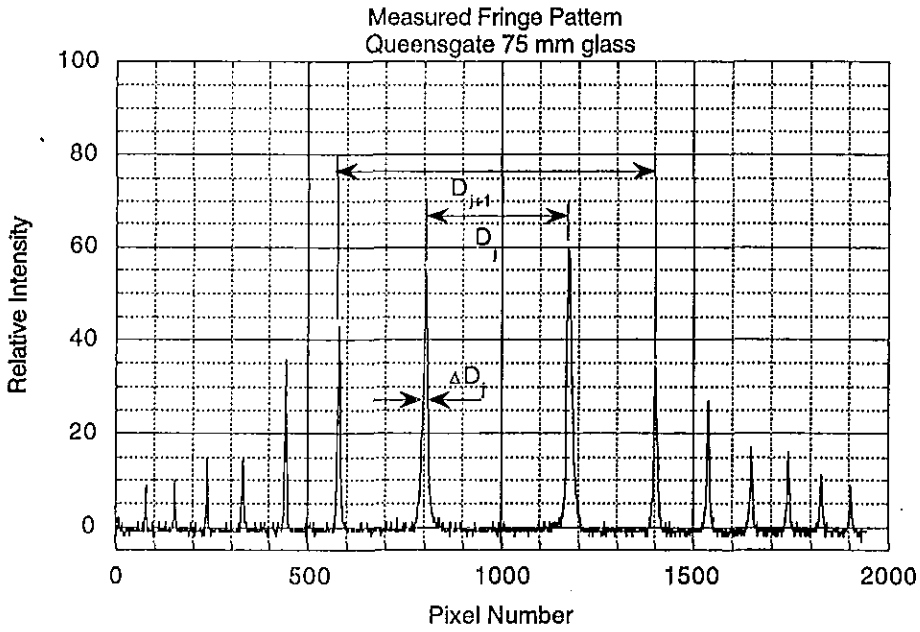
The finesse of a given fringe pattern may be found by using the following equation:

$$F = \frac{D_{j+1}^2 - D_j^2}{4D_j \Delta D_j} = \frac{D_j}{4j\Delta D_j}$$

where

F = finesse
D_j = fringe diameters
ΔD_j = FWHM
j = fractional order (Section 7.1.1)

In the following example, $j \approx 0.25$ and the finesse $F \approx 26$:



Notes:

1) Our fixed etalons generally have finesses on the order of 20 or so. The tunable Burleighs, however, may attain finesses as high as 50 or 60.

2) As a rule of thumb, the finesse of the etalon is approximately equal to the number of bounces that the light undergoes inside the etalon.

Even if the mirrors are perfectly flat and perfectly parallel, the etalon will have a finite finesse determined by the reflectivity of the mirrors. This is called the reflectivity finesse and is given by:

$$F = \frac{\pi\sqrt{R}}{(1-R)}$$

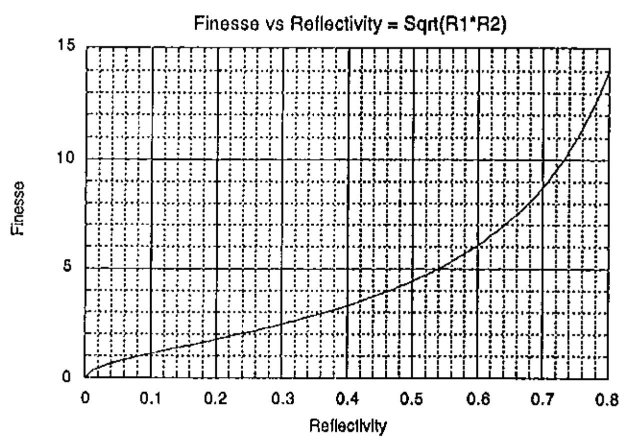
where

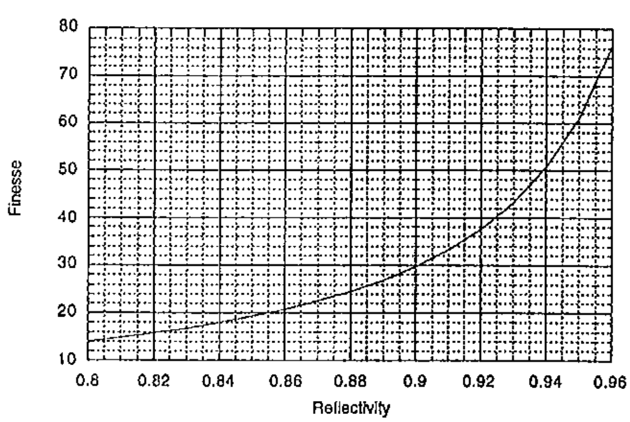
$$R = \sqrt{(R_1 * R_2)}$$

F = reflectivity finesse
R_1 = reflectivity of front mirror
R_2 = reflectivity of back mirror

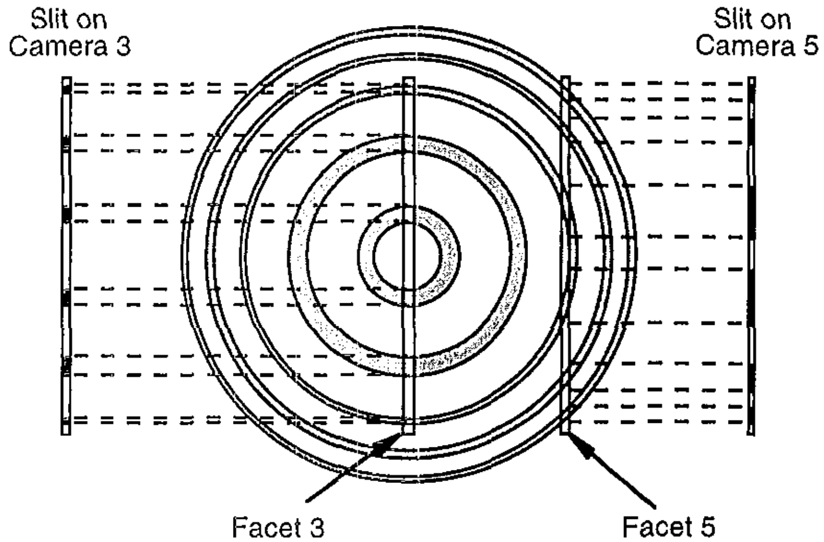
The actual finesse of an etalon will almost certainly be less than the value given by the above equation. There is a nice discussion of the effects on finesse of different types of mirror defects in the following reference: "The Fabry-Perot Interferometer", JM Vaughan, 1989, pp 104, 123-127.

The following graphs give reflectivity finesse as a function of R:





The geometry of the analyzer table does not allow the full finesse of the etalon to be realized at all of the streak cameras. At the 5-facet mirror, the fringes are formed by the intersection of the Fabry rings with the vertical columns of light allowed by the table optics. These fringes are then relayed to the streak camera slits.



Note for facet 3, the vertical column intersects the Fabry rings near the diameter where the rings are nearly horizontal. This results in fringes with widths that are nearly the same as that allowed by the finesse of the etalon. For facet 4, and even worse for facet 5, the vertical column intersects the rings at a chord where the rings have a large vertical component. This results in fringes with greater widths, which results in a lower finesse value.

The full width of the fringes at the 5-facet mirror relays to fringe widths of 137 μm at the streak camera slits. Using narrower slits improves the finesse at large chord distance in the ring pattern. The following table calculates the finesse realized at the streak cameras for different slit widths assuming an etalon finesse of 20.

	Slit Widths (μm)				
Facet #	137	100	75	50	25
3	20.0	20.0	20.0	20.0	20.0
2,4	18.4	19.1	19.5	19.8	19.9
1,5	15.3	17.0	18.2	19.1	19.8
(6)	12.2	14.5	16.3	18.0	19.4

Notes:

1) There is not much improvement in finesse for slit widths less than 75 μm.

2) The last row calculates the finesse for a fictitious facet #6 to show the further degradation of finesse at large chord distance. This effectively limits the analyzer table to five cameras.

As light enters an etalon, there are two competing effects. Some of the light gets trapped inside the etalon and bounces back and forth between the mirrors. On the other hand, some of the light is lost upon each reflection from the mirrors. If light is continuously incident upon the etalon, the amount of light inside the etalon will increase until the rate at which the light is trapped will essentially equal the rate at which the light is lost. At this point, we say that the etalon is filled and the finesse has reached its final value. The amount of time that it takes to fill the etalon gives an indication of the time response of the etalon. Mathematically, the etalon never fills, but approaches an asymptote. We use 99.99% filled to calculate the fill time. The fill time τ may be calculated from the following equations:

Air cavity:

$$\tau = \frac{2H}{c} \frac{\ln\left[1-\sqrt{(\%\text{filled})}\right]}{\ln(R)}$$

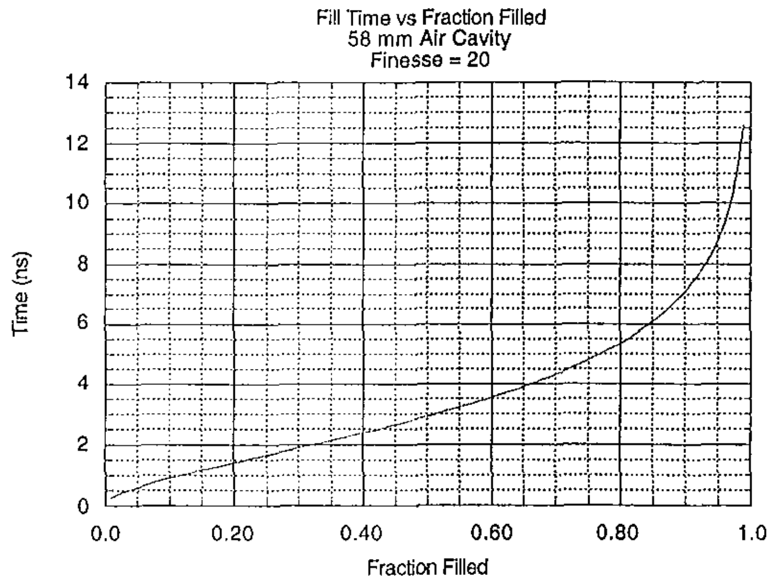
Glass cavity:

$$\tau = \frac{2nT}{c} \frac{\ln\left[1-\sqrt{(\%\text{filled})}\right]}{\ln(R)}$$

where

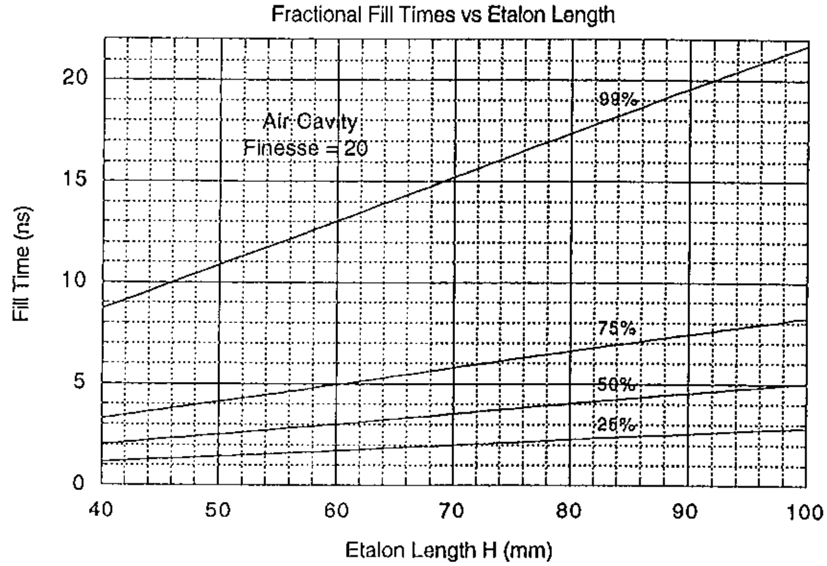
H = length of air cavity
T = length of glass cavity with index of refraction n
c = speed of light in air
$R = \sqrt{R_1 * R_2}$ and R_1, R_2 = reflectivity of the front and back mirrors.

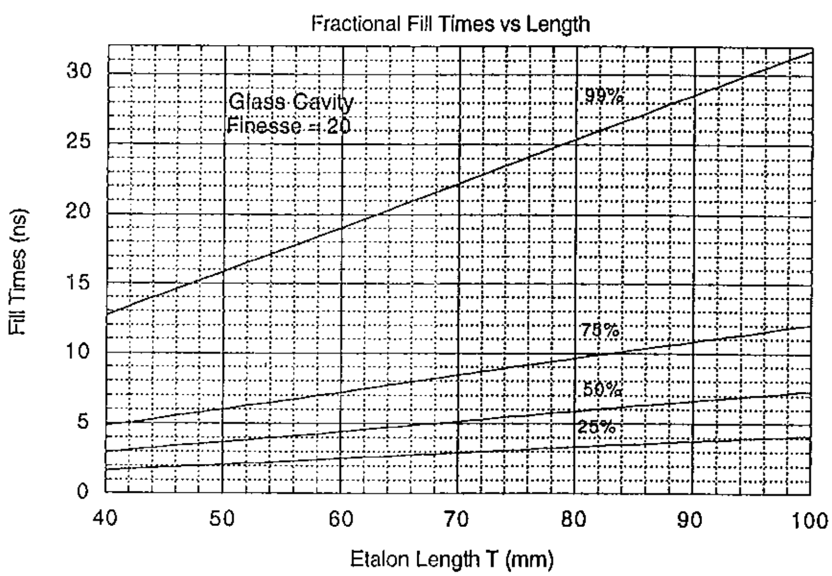
The following graph shows the etalon fill time vs the fraction filled for an etalon with a reflectivity finesse of 20. A value of R = 0.855 yields a reflectivity finesse of 20 (Section 4.6.5).



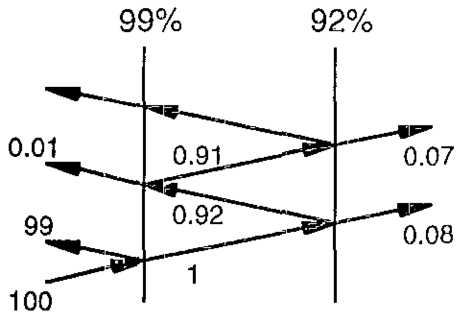
Note: The above equations do not include other factors, such as walkoff, which also affect fill time.

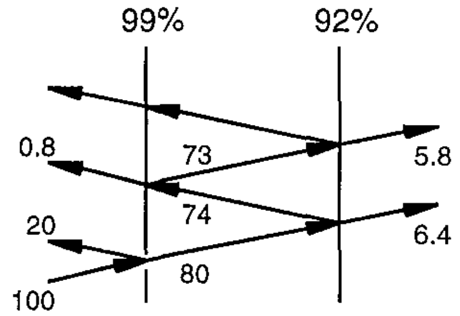
As an etalon fills, the finesse improves as the number of bounces increases. When measuring, say, shock arrival at high time response, it is possible to see the fringe widths decrease as the etalon fills at the new wavelength. An etalon does not need to be fully filled for the fringes to be formed. Indeed, it is possible to follow the velocity time history of an experiment in which the data changes so fast that the etalon has time to fill to only a fraction of its final fill. Ed Daykin has done some time-dependent modeling of fringe formation which suggests that fringes become useable for data analysis at 30% to 50% fill. The following graphs show the times for an air cavity and a glass cavity to fill to 25%, 50%, 75%, and 99% of final fill vs the etalon lengths.





Consider light encountering a set of mirrors with $R_1 = 99\%$ and $R_2 = 92\%$. The sketch on the left shows that 99% of the light reflects off the front mirror and is lost. Suppose the mirror coating were removed in a small area to allow, say, 80% of the light in, as shown in the sketch on the right. Much more light enters the etalon, and much more light exits the back mirror. On our analyzer tables, the light from the fibers is imaged to a horizontal line, and a narrow stripe of the mirror coating is removed from the front mirror to allow more light to enter the etalon.





Note: Gaining extra light with a striped etalon is not without its disadvantages:

- the striping process is very tedious
- striping adds to the overall cost of the etalon
- low-order fringes are lost in the data records
- preparing the light to enter the stripe consumes more room on the analyzer table.

In fact, there are not many other labs that use striped etalons.

Having the stripe in the front mirror means that very low-angle light will escape out the stripe without being captured by the etalon. At an entrance angle of zero degrees, nearly all of the light exits the stripe after one bounce off the back mirror, therefore, no fringes are formed at zero degrees. As the entrance angle increases, a larger fraction of the light is captured by the etalon until, at some angle, all of the light is captured. The fringe brightness gradually increases with increasing angle until some angle beyond which all fringes have the full brightness allowed by the length of the etalon and the width of the stripe. The quantity p_{st} is defined as the fringe order at which the fringes attain full brightness allowed by the stripe width and etalon length. The next page uses p_{st} to calculate stripe widths.

Determining stripe width: There have been two different methods suggested to determine strip width:

1) An algorithm for selecting the stripe width has been suggested* that sets $p_{st} = 4$, that is, a fringe at an order of j = 1 will have half the brightness of a fringe at j = 4:

For an air cavity with length H: $$w_{st}(mm) = 4\sqrt{\lambda H} = 0.0923\sqrt{H(mm)}$$

For a glass cavity with length T: $$w_{st}(mm) = 4\sqrt{\frac{\lambda T}{n}} = 0.0764\sqrt{T(mm)}$$

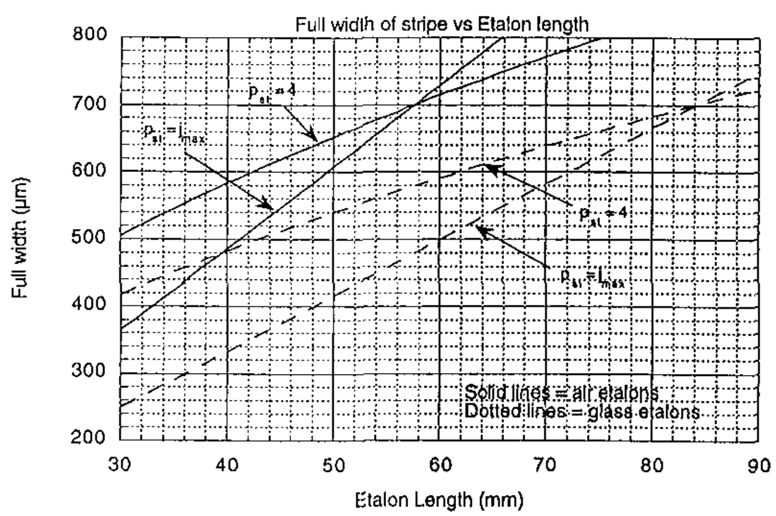
where w_{st} = the full width of the stripe
λ = laser wavelength = 532.1 nm
n = index of refraction of the glass cavity = 1.46071

2) Many of our new etalons are short enough that the fringe order j = 4 is not recorded by the streak camera. More recently, Ed Daykin has suggested an algorithm that sets $p_{st} = j_{max}$, that is, a fringe at an order of j = 1 will have half the brightness of the maximum fringe order recorded by the streak camera. This has the advantage of putting more light into low fringe angles for short etalons. Interestingly, this algorithm is independent of λ:

For an air cavity with length H: $$w_{st}(mm) = 2 * \theta_{max} H = 0.0115 * H(mm)$$

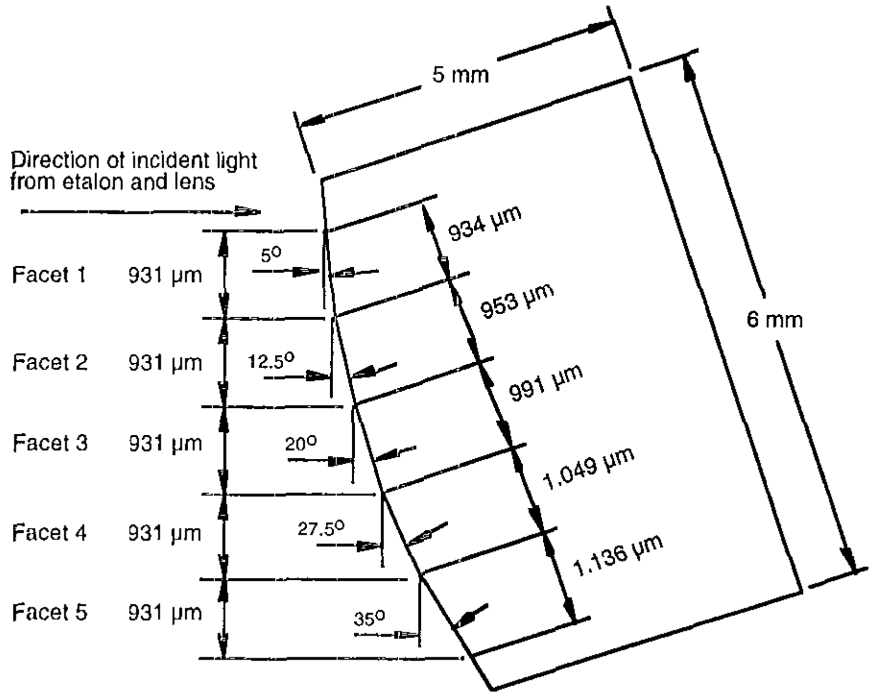
For a glass cavity with length T: $$w_{st}(mm) = 2 * \frac{\theta_{max}}{n} T = 0.00789 * T(mm)$$

where θ_{max} is discussed in Section 4.6.3.



* "Formulas for Fabry-Perot velocimeter performance using both stripe and multifrequency techniques," David R. Goosman, Applied Optics 30 (27), p. 3912.

The reflective separator (5-facet mirror) is a set of 5 tall thin mirrors that are ground into the side of a single piece of gallium gadolinium garnet (GGG). (Garnet is a class of hard vitreous silicates doped with some mineral.) The substrate is coated with aluminum for the mirror surface, and then protected with a hard transparent silicate overcoat. The 5-facet mirror is placed at the focal plane of the spherical mirror (after the etalon) and is used to reflect the set of fringes from each fiber into its own streak camera. The entire 5-facet mirror is angled so that the light to camera 1 is reflected at an angle of 10 degrees. The angle of each facet is different by 7.5 degrees so that the streak cameras are separated by 15 degrees around the analyzer table. Each mirror facet is approximately 1 mm wide by 28 mm tall. The actual width of each facet is slightly different so that its projection in the direction of the etalon is 931 μm wide. This piece of optic is difficult to make accurately and is quite expensive. The following sketch shows the top view of the 5-facet mirror.



Note: Facets 1 and 5 actually extend to the edge of the glass. We have used this extra width to duplex camera 5 with the 6-fiber chuck (section 4.3).

Section 4.8

Analyzer Table
Table Tune

The following steps should be done prior to a shot if the analyzer table has not been used in quite some time. The next section (4.8.2) has the steps to perform every morning thereafter to assure that the alignment has not drifted.

Note: We use the following coordinate system on the analyzer table: z is in the direction that the light travels, x is left/right, and y is up/down.

Bare beam

-Push the fiber chuck, shaper, pickup lens, and cylinder lens off to the side. Also take out the bandpass filter.
-Send light to the bare beam set of mirrors.
-Use the combination square to check the alignment of the bare beam at each end of the first leg. It doesn't hurt to repeat this check periodically during the remaining bare beam steps.
-Check the height of the etalon by holding a white card at the output end of the etalon and making sure the stripe bisects the bare beam.
-Check the retro of the etalon. Go back to the previous step and recheck the etalon height.
-Put the combination square at the far end of the first leg.
-Insert the pickup lens and check alignment at the combination square.
-Insert the cylinder lens along with the pickup lens and check the height at the combination square. Then remove the cylinder lens.
-Insert the shaper along with the pickup lens and check at the combination square. If the beam is now off-target, the shaper needs some adjustments.

> -The following steps assume that the shaper had been properly set originally and now needs only slight adjustments in x/y and tilt.
> -Put the combination square in between the shaper and the pickup lens. You will need to set it on a thin brass block and make a new mark on the square with the bare beam.
> -Put the square as close to the shaper as possible and adjust tilt (pitch and yaw).
> -Put the square as close to the pickup lens as possible and adjust x/y.
> -Repeat as many times as it takes. This will take a while--be patient.
> -Occasionally put the square at the end of the 1st leg and recheck.

-Insert target into the front snout of the shaper and note location of the bare beam. It may not be exactly centered, but that is okay.

Fiber chuck

-Insert the fiber chuck and send light to the center fiber.
-Note the approximate z position of the chuck. You will want to return the chuck to this position after the following two steps.
-Move the chuck forward in z (toward the shaper) until the tip almost touches the target. Translate x and y to the bare beam location--not the center of the bulls-eye.
-Move the chuck away from the shaper until the spot of light fills the outer ring. Adjust pitch and yaw until the spot is centered. Iterate this step and the previous step until the fiber chuck is aligned to the target. Remove the target from the snout of the shaper.
-Return the chuck to its original z position.
-Use the 5-hole card to check the alignment of the fiber chuck, shaper, and pickup lens.
    -First, look at the center hole and adjust the fiber chuck in x/y until aligned.
    -Second, look at the holes at 3 and 9 o'clock. Adjust the pickup lens in z to align.
    -Third, look at the holes at 12 and 6 o'clock. Adjust the fiber chuck in z to align.
    -Iterate the previous three steps until all 5 holes are aligned.
-Check the roll orientation of the fiber chuck with the following steps
    -Insert the cylinder lens
    -Set the combination square immediately in front of the etalon
    -Put light into fiber #1 and check the height of the line focus in front of the etalon
    -Put light into the last fiber at the other end of the chuck.
    -Check the height of the line focus in front of the etalon.
    -Adjust the fiber chuck in roll until the line focus is the same height for all fibers.
-Recheck the 5-hole card with the cylinder lens out.
-Insert the bandpass filter, check that the reflection is just next to the output of the shaper.

Cylinder lens

-Insert the cylinder lens
-Check the roll orientation of the cylinder lens with the following steps
    -Set the combination square immediately in front of the etalon
    -Put light into the center fiber
    -Check the height of the line focus along its entire length from -x to +x.
    -Adjust the cylinder lens in roll until the height is the same over the full length.
-Insert a white after the etalon and before the spherical lens. Adjust the height of the cylinder lens to obtain approximately an even amount of light top and bottom. This is only a course height adjustment for the cylinder lens.

(Note: if you have a tunable etalon, you will need to start maintaining a good tune here.)

5-facet mirror

-Place a sheet of lens tissue in front of the primary and referee etalons.
-Light is still in the center fiber only. Block the light to the referee cavity.
-Check that the primary rings are in focus at the 5-facet mirror.
    -Focus is adjusted with the z position of the spherical lens
-Let light go to the referee cavity and check concentricity (this step is for a roundabout).
    -Concentricity is adjusted at the recombination mirror.
    -Note: for an air cavity with a glass insert, the rings should be concentric when the z of the spherical lens is correct--this is actually a good way to set the ring focus.
-Take out the lens tissue.
-Put light in all five fibers.
-Check that the fringes are co-linear at the 5-facet mirror (this step is for a roundabout).
    -Co-linearity is adjusted at the pick-off mirror.
-Set the combination square immediately in front of the 5-facet mirror.
-Check the height of the fringe pattern. Adjust spherical lens in y.
    -Note: for an air cavity, the pattern may be off in x if the mirror wedges were not compensated properly. This is not critical.
-Check visually to see that the fringes are falling on the facets.
-Use a white card to look at the reflections from the facets. Verify that light is going to all five cameras. Move the 5-facet mirror in x to set the fringes approximately in the center of the facets.
-Set a white card in front of each relay lens and check balance. Move the cylinder lens in y to adjust.
-Check that the light is centered in the relay lenses. Adjust rotation of 5-facet mirror.
-Remove white cards

Streak cameras

-Verify that the cameras are positioned correctly in z with the following steps.
    -Place a target at the 5-facet mirror with some form of pattern on it.
    -Use a gooseneck lamp or a fiber with bright 532 nm light to illuminate the target
    -Look at the target using the slit monitor for each camera.
    -Adjust each camera in z to achieve best focus.
    -Secure each camera to its rail.
-Turn up the laser to check that the fringes are centered in the facets. Look at the slit monitor and verify that the pedestal is in the center of the visible portion of the rings. If the rings are not visible, try putting a sheet of lens paper in front of the etalons.
-With the laser turned up, make SMALL z adjustments to the pick-up lens to minimize the width of the pedestal.
-Adjust each relay lens in y to center the fringe pattern vertically on slits.
-Fringes should be co-linear and in focus at the slits.
-Clear all unnecessary items from the table for the shot.

This page contains an excerpt from the daily checklist that we use at U1a leading up to a shot. This list assumes that the full alignment given in section 4.8.1 has already been performed recently.

Note: We use the following coordinate system on the analyzer table: z is in the direction that the light travels, x is left/right, and y is up/down.

Timing check

-Download settings for FTGs.
-Verify all trigger times, including the laser trigger.
-Check streak camera sweep settings and gains.
-Trigger system.
-Verify timing using polaroids.

Analyzer table checklist

-Launch light into the center fiber of the fiber chuck.
-Take out the bandpass filter.
-Check the retro with the 5-hole card. If this is good, go to the next steps.
    -If the chuck or the pickup lens needs slight z-adjustments, this is okay to do.
    -If all spots are off in the same direction, go back to the bare beam to check the etalon retro.
    -If the etalon retro is good, then the fiber chuck needs to be adjusted in x/y.
-Insert the bandpass filter, check that the reflection is just next to the output of the shaper.

(Note: if you have a tunable etalon, you will need to start maintaining a good tune here.)

-Insert the cylinder lens.
-Check the coarse balance with a white card after the etalon.
-Check that the Airy rings are concentric and in focus at the 5-facet mirror.
-Put light in all five fibers.
-Check that the fringes are co-linear at the 5-facet mirror.
-Check the balance at the relay lenses.
-Check that the light is centered in the relay lenses.
-Using the slit monitor, turn up the laser to check that the fringes are centered in the facets.
-Check the height of the fringe pattern on the slits.
-Check the focus and alignment of the fringes at the slits.
-Clear all unnecessary items from the table for the shot.

(This section will grow as I receive inputs. Let me know if you have a favorite solution to a common alignment problem on the analyzer table.)

Problem: Distorted fringes at the streak camera slits.

Things to check:
1) Basic alignment. Verify fiber/shaper/pick-up optics with 5-hole card using steps given in the full alignment section 4.8.1.
2) Camera focus. Use steps given at bottom of p. 3, section 4.8.1 to verify camera z. This is a good candidate if only one or two cameras have a problem; if all the cameras have a problem, the solution probably lies somewhere else.
3) Alignment at relay lenses. The bundle of rays leaving the 5-facet mirror should pass through the centers of the relay lenses. Adjust tilt of the 5-facet mirror.
4) Spatial variations in etalon quality. Use an opaque card and block some of the light entering the etalon. Start at the left end of the stripe and move the card slowly inboard, then repeat from the right end. You may end up permanently blocking some of the light.
5) Distortions in slit monitor. It is possible that the fringes are actually okay, but the image at the monitor is distorted. Remove both covers from the foreoptics and adjust the relay mirrors from the slit to the Cohu camera. Also check the focus of the Cohu camera.

Problem: Fringe brightness is not balanced top vs bottom on all five streak camera records.

Most likely thing to check: the height of the focus at the etalon stripe. Perform the following steps:
1) Put the combination square directly in front of the etalon. Check the height of the focus. If this is not exactly on the mark, adjust the height of the cylinder lens.
2) Adjust the height of the etalon (light in all five fibers). People have several favorite places to monitor this adjustment:

a) A small white card directly in front of the 5-facet mirror. (Be careful not to scratch the mirror.)

b) A 3 x 5 white card in front of each relay lens.

c) Slit monitor--be sure to turn down the light from the laser so that the fringes are not saturated on the monitor. (I recommend method a) or b).)

Note: If the table has a roundabout, block the light from the referee and adjust the primary cavity height. Then block the light from the primary and adjust the referee.

Problem: Fringe brightness is not balanced top vs bottom on only some of the cameras.

Things to check:
1) Tilt of the cylinder lens. Perform the following steps:
   a) Put light in only the center fiber.
   b) Put the combination square directly in front of the etalon.
   c) Slide the combination square left and right along the focus of light and make sure the height is constant from one end to the other.
   d) If not, adjust tilt (roll) of the cylinder lens.
2) Roll of the fiber chuck. Perform the following steps:
   a) Put the combination square directly in front of the etalon.
   b) Put light in only fiber 1. Check height of focus at the etalon.
   c) Put light in only the last fiber (at the other end of the fiber array). Recheck height.
   d) If not equal, adjust roll of the fiber chuck. You may need to iterate several times.
3) Fibers not co-linear in fiber chuck. There is no adjustment for this, but use the following steps to check:
   a) Slide the etalon out of the beam path.
   b) Move the cylinder lens out of the beam path.
   c) Put a small white card at the 5-facet mirror.
   d) Put light in all five fibers.

The light from the five fibers will focus to five "footballs". If the footballs are not co-linear, then the fibers in the chuck are not co-linear, either. You will either have to live with this, or put a new fiber chuck on the table. I recommend a chuck built with the silicon v-grooves.

Section 4.9

Analyzer Table Duplexing

There are times when an experiment requires more velocimetry data channels than are available using the standard geometry. Normally, a data channel is composed of one probe and one streak camera. It is possible, however, to record the data from two probes onto a single streak camera. We call this 'duplexing'. Duplexing effectively doubles the number of data channels that may be recorded for a given number of streak cameras.

The sketch in Section 5.5 shows a fictitious data record using the standard one-probe-per-camera technique. Note that the upper half of the record is a mirror image of the bottom half. Strictly speaking, then, the bottom half is redundant and could be replaced by the bottom-half data from a different probe. The optics on the analyzer table allow this to happen, so that the light from two probes may be recorded onto a single streak camera. The light from one probe would be recorded on the upper half of the data record, and the light from a second probe would be recorded on the bottom half of the data record.

Advantages:

1) There is only one advantage to duplexing--more data channels.

Disadvantages:

1) Decreased velocity and timing precision.
2) Increased bookkeeping demands.
3) Increased likelihood of confusion during set up.
3) Additional optics on the analyzer table, including fiber chuck.
4) Tighter tolerances in the alignment of the 5-facet mirror.
5) An additional streak record must be made just prior to the shot to determine fringe centers.
6) Data analysis requires more effort.
7) The loss of a data record means the loss of two channels of data.

Notes:

1) Duplexing is still in its infancy. At this writing, one or two cameras per table have been duplexed at U1a on three occasions, and all five cameras per table have been duplexed at Site 300, B851, on two occasions. No data has been lost by duplexing.

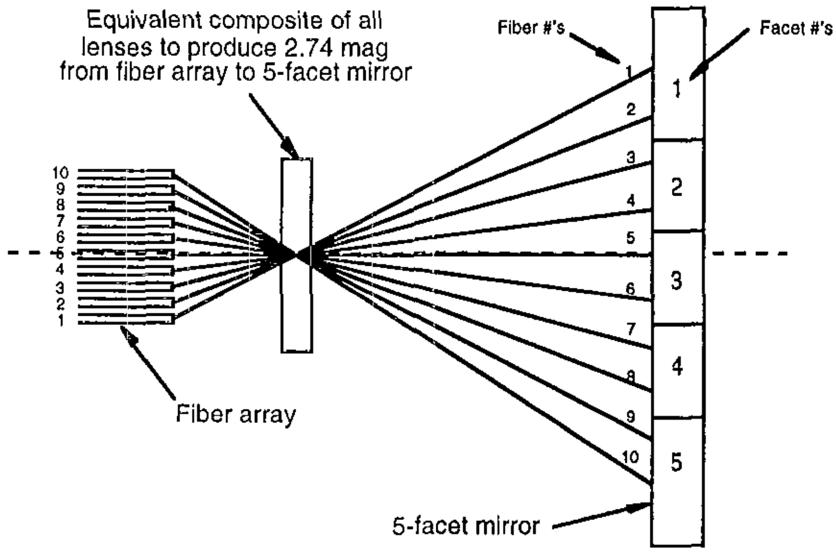
2) The single biggest hindrance to duplexing has been the lack of fiber chucks with enough fibers (see Section 4.3 for a description of fiber chucks). For example, all of our analyzer tables originally had the 6-fiber chuck, which allows only camera 5 to be duplexed. More recently, several of our tables now have 10-fiber or 11-fiber chucks, or chucks with silicon v-grooves and12 fibers, which allows all five cameras to be duplexed.

3) Even after duplexing was announced as a viable means to obtain data, no designer wanted to be the first to have his data recorded that way. It took considerable effort by Rex Avara and Tony Rivera to quantify the uncertainties involved with duplexing and to develop a streak camera calibration technique to support it.

4) Avara, Collins, and Rivera wrote a very complete description of streak camera uncertainties for the HHPs in "Distortion Corrections for the Many Beam Fabry Perot Velocimeter," UCRL-ID-143225, April 2001.

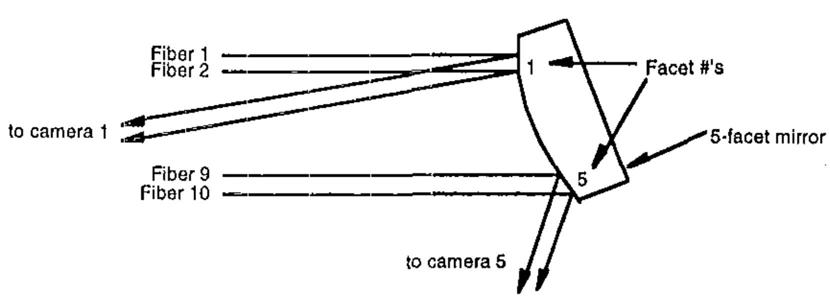
A basic discussion of the analyzer table optics is given in Section 4.2, but a closer look is helpful here to understand how the duplexing technique works.

Looking from a top view, the fibers in the array are imaged to the 5-facet mirror with a magnification of 2.74. Our original 6-fiber chucks have a fiber-to-fiber spacing of 340 μm, which map one fiber onto each of the 930-μm-wide facets. On the other hand, if the fiber array had 10 fibers with a spacing of 170 μm, then two fibers would be imaged onto each facet. The images of the fibers will be approximately 465 μm apart at the 5-facet mirror.



With a 10-fiber chuck, fibers 1 and 2 would be imaged onto facet 1, fibers 3 and 4 would be imaged onto facet 2, and so on. In the set-up shown here, fiber 5 is aligned on the optical axis, that is, light is launched into fiber 5 when using the 5-hole card to check the table alignment. Any fiber near the center of the array may be positioned on the optical axis. The 5-facet mirror needs to be aligned so that the proper set of images falls on each facet.

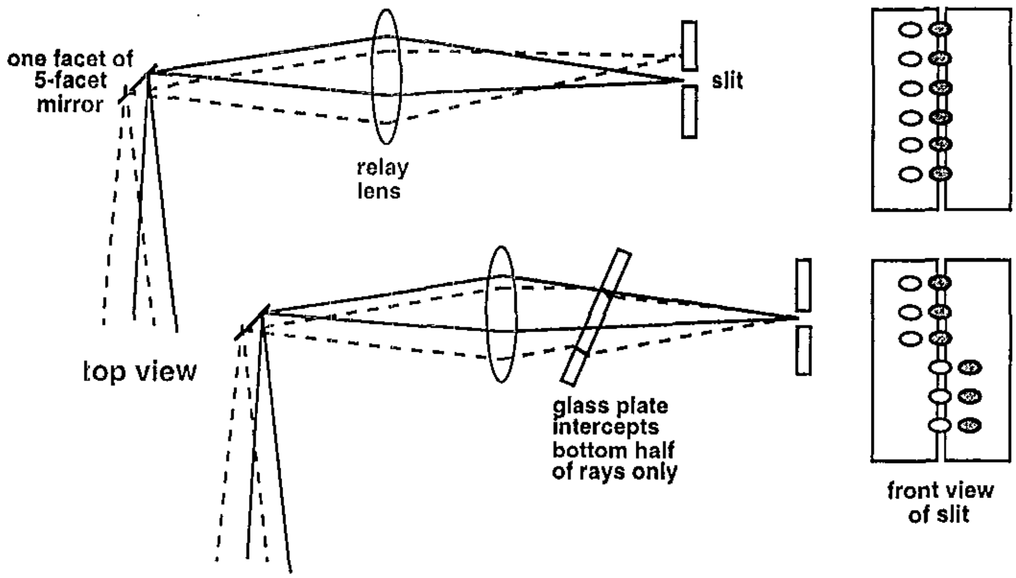
Because each facet is angled 7.5 degrees with respect to its neighbors (Section 4.7), the light from each set of fibers will be reflected 15 degrees apart toward each streak camera (see the sketch in Section 4.1). Only the reflections from facets 1 and 5 are shown below for clarity. The relay lenses transport these images to the streak camera slits with a 2:1 demagnification, so that the images at the slits are approximately 233 μm apart.



Now that the fringes from two probes are at the slit of a single streak camera, the next two sections show how to duplex either a single camera or all five at once.

Section 4.9.2 describes how a 10-fiber chuck will image the light from 2 fibers onto each facet of the 5-facet mirror. The top part of the sketch below shows the imaging from one of those facets to the streak camera slit. The result is two columns of fringes, one from each probe, approximately 233 μm apart at the slit. Use the relay lens to position one set of fringes onto the slit as you normally would.

If only one or two extra data channels are required on a shot, the duplexing is performed in the optical leg between the 5-facet mirror and the streak camera. The bundle of rays in this leg have its largest lateral extent at the relay lens, so this is the easiest place to duplex. A thin glass plate is inserted into the bottom half of the bundle of rays just behind the relay lens. Rotating this duplexing plate about a vertical axis will translate the bottom set of fringes as described in Section 8.2. Rotating in the correct direction and by the correct amount will move the bottom set of fringes so that the fringes from the other probe will enter the slit.



Notes:

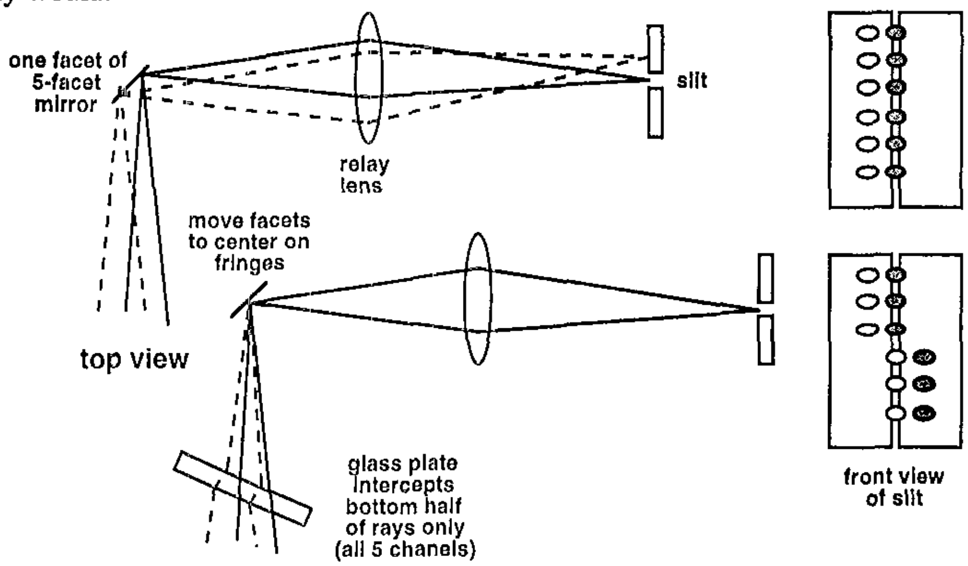
1) It is important to use a fairly thin duplexing plate to avoid significantly extending the focal plane of the bottom set of fringes (Section 8.2).

2) If only one camera is duplexed, it may be easiest to duplex camera 1 or 5 because the outermost facets in the 5-facet mirror are the widest ones.

3) If two or more cameras are duplexed, the alignment of the 5-facet mirror becomes critical because the magnification from the fiber array to the 5-facet mirror is not always perfect. In this case, it may be easiest to duplex adjacent cameras because the lateral displacement of adjacent sets of fringes caused by a magnification error is minimized.

4) At the 5-facet mirror, the fringes are near the edges of the facets in this geometry. The left/right alignment of the 5-facet mirror is more critical than usual when duplexing at the relay lens.

Section 4.9.2 describes how a 10-fiber chuck will image the light from 2 fibers onto each facet of the 5-facet mirror. The top part of the sketch below shows the imaging from one of those facets to the streak camera slit. The result is two columns of fringes, one from each probe, approximately 233 μm apart at the slit. Switch through each of the streak camera slit monitors and use each relay lens to position, say, the left set of fringes onto each slit as you normally would.



The easiest way to duplex all five cameras is to place a thin glass plate directly in front of the 5-facet mirror. This duplexing plate should be positioned at a height to intercept only the bottom half of the bundle of rays. Rotating the duplexing plate about a vertical axis will translate the bottom set of fringes as described in Section 8.2. Rotating in the correct direction and by the correct amount will move the bottom set of fringes so that the fringes from the other sets of probes will enter the slits. Reposition the 5-facet mirror left/right so that the fringes being imaged to the slits are centered in each facet. Once again, switch through the slit monitors for all five cameras to ascertain that the alignment is correct for all sets of fringes. Several iterations of plate angle and relay lens positioning may be needed to optimize the alignment for all five cameras.

Notes:

1) It is important to use a fairly thin duplexing plate to avoid significantly extending the focal plane of the bottom set of fringes (Section 8.2).

2) The fringes actually imaged to the slits are nearly centered in each facet of the 5-facet mirror. The left/right alignment in this geometry is actually easier than in the case of duplexing only one or two cameras.

3) It is very important to verify probe number with up/down position on each piece of film. Unplug, say, the even numbered probes and fire a local dry run on polaroid film just to make sure. Carefully label each piece of film and save for shot day and later analysis.

4) Consider putting probes with slightly different jump-off times together on a camera so that causality will help minimize confusion in data location vs probe number.

4.9.5 Analyzer table-Duplexing
Data Analysis

In our normal streak camera records, the data is symmetric about a horizontal line in the middle of the fringe pattern. That is to say, the upper half is a mirror image of the bottom half. The figure on the last page of Section 4.6.5 shows that the fringes are located at the intersections of the streak camera slit with the Airy rings. All of our data analysis relies upon being able to measure the distance between a fringe in the upper half and its counterpart in the lower half. The figure in Section 7.1.1 shows how the "diameters" are measured in our standard data analysis.

Unfortunately, when we duplex a camera with data from two different probes, the upper half of the streak camera record is no longer a mirror image of the bottom half. The time history of the upper half may be completely different from that in the bottom half, and we may no longer simply measure diameters to perform the data analysis. In the case of duplexing, the center of the fringe pattern must be determined, and then the "radii" are used. In the figure in Section 7.1.1, the dotted horizontal line represents the fringe center. The radii from the dotted line to the upper set of fringes would be used to analyze the data from one probe, and the radii from the dotted line to the bottom set of fringes would be used to analyze the data from the other probe. The equation at the top of Section 7.1.1 still applies with the radii used instead of the diameters.

When duplexing, then, it is very important to determine the fringe center for each camera. This must be done just before the shot and after all analyzer table adjustments have been completed. To determine the fringe center, carefully remove the duplexing plate and fire a local dry run using hard film. Replace the duplexing plate and verify that both the upper set and lower set of fringes are aligned with the streak camera slit. Re-load the camera with fresh hard film. You are now ready for the shot.

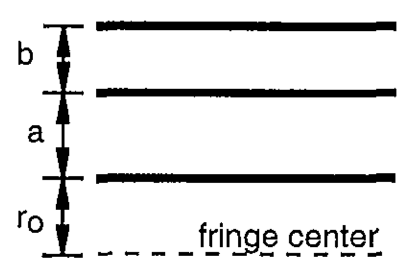
For the data analysis, our processing software has routines to handle the duplexed data. The piece of film exposed without the duplexing plate is analyzed first. The upper and lower fringes are symmetric on this piece of film and the fringe center is found. The center is then precisely located with respect to the timing fiducials. The shot film is then analyzed and the fringe center is inserted. Two new data files are then created--one with the upper set of fringes and its mirror image, and the other with the bottom set of fringes and its mirror image. The timing fiducials are replicated in each of the new files. Each new file is then analyzed as a regular piece of data where the "diameters" are actually twice the "radii".

Notes:

1) There is some loss of velocity accuracy with duplexing. Avara, Collins, and Rivera wrote a very complete description of streak camera uncertainties for the HHPs in "Distortion Corrections for the Many Beam Fabry Perot Velocimeter," UCRL-ID-143225, April 2001.

2) Rex Avara has derived a formula to determine the fringe center at a single timepoint using the distances between two adjacent fringes:

$$r_o = \frac{ab}{a-b} - \frac{a+b}{2}$$



Section 5

Streak Cameras

An entire handbook could be written on just streak cameras. The cameras are sophisticated and complicated electronics; only a knowledgeable person should attempt to perform maintenance or modifications to the cameras themselves. Fortunately, the operator of an analyzer table does not need to know very many details of the camera operation. This handbook covers only those topics that are useful to the operator on a normal day of setting up and operating the analyzer table.

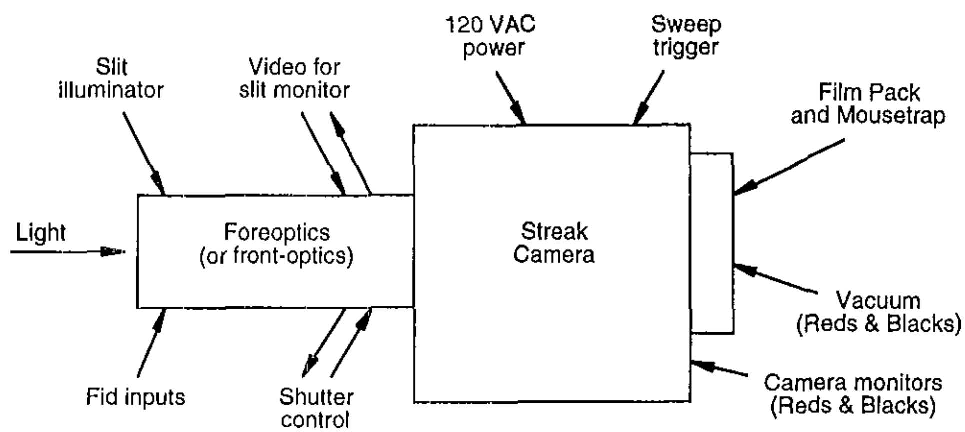
We have three different types of streak cameras in use. They are different in their details, but they all have many features in common:

HHP—These cameras were designed and built at Livermore. They use Hamamatsu tubes (model #N2214) with an S20 input photocathode and a P11 output phosphor. These cameras are used at HEAF and at Site 300.

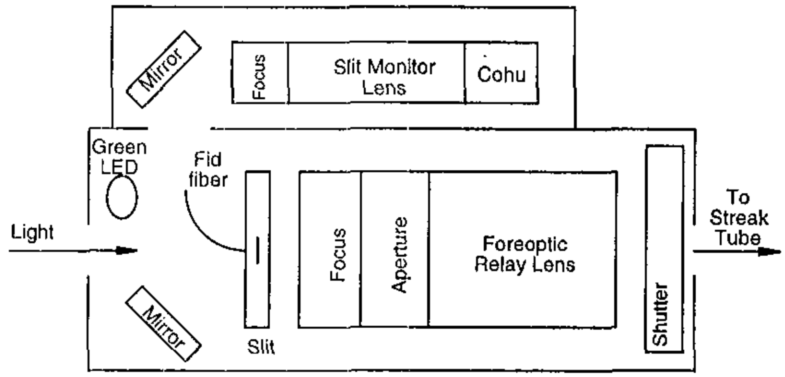
EGG Red—These cameras (L-CA-18) were originally designed and built by EGG for the underground nuclear test program and are approximately 20 years old. They originally had nanosecond sweeps, but the sweep boards were modified to slow the sweeps to the microsecond range needed for hydro tests. They use ITT tubes with an S20 input photocathode with an enhanced sensitivity in the red region of the spectrum (hence the label "red", even though the chassis housings are also red) and a P11 output phosphor. There are only six of these cameras and are used at U1a. These cameras use vacuum to pull the film up against the MCP.

Bechtel Black—These cameras (LOE-97-100) were built by Bechtel Nevada specifically for use on hydro tests at U1a and at BEEF. These cameras are very similar to the Reds in their physical appearance (except they have black housings) and in their operation. They have new streak tubes designed by Bechtel and other improvements based upon experience gained at U1a. The tubes have an S20 input photocathode and a P11 output phosphor. These cameras use vacuum to pull the film up against the MCP.

The following figure shows the basic parts of our streak camera systems and the functional inputs and outputs required.



The foreoptics box is mounted to the front of the streak camera. It accepts the light in the form of fringes from the analyzer table and relays those fringes to the photocathode of the streak camera. The main components of the foreoptics are the slit assembly, the relay lens, the shutter, and the slit monitor.



Slit Assembly: The slits are made from two razor blades positioned with their sharp edges adjacent and parallel to each other. The gap between the blades makes the slit through with some of the light from the fringes pass. The slits are 11 mm high by either 50 μm or 100 μm wide. The blades are held by an assembly that permits adjustments of the slit position and orientation. The fibers that send the fids to the camera are mounted on the slit assembly, also. The fibers are held 12 mm apart at each end of the slit.

Relay lens: This set of lenses (Micro-Nikkor, 55mm, f/2.8) focuses the fringes from the slit onto the photocathode of the streak tube. The relay lens has adjustments for focus and aperture. The lens rides on an assembly that allows the lens to be moved closer to the slit or closer to the streak tube; this adjusts the magnification of the fringes at the photocathode and on the film.

Shutter: The shutter is made by Uniblitz Vincent Associates, Rochester, NY. The shutter is opened by remote control for only a second or less at shot time to prevent excess room light from entering the camera and exposing the film. The shutter on each streak camera is interlocked to prevent the shot from firing in the event that one of the shutters fails to open.

Slit Monitor: The foreoptics also has a box mounted on its side that contains the Cohu slit monitor camera to view the fringes on the slit. There is a set of two mirrors to send the image of the slit to the monitor camera. A green LED is mounted inside the front of the foreoptics box to illuminate the slit for the monitor camera.

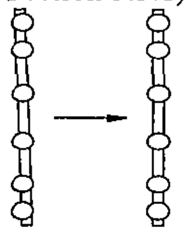
Most of the time that an analyzer table operator spends on the streak camera system will be spent on the foreoptics box. The following are some of the common adjustments required on the foreoptics. There are other adjustments to make when the foreoptics are first mounted onto the streak camera (Section 5.3.1):

1) Align the image of the slit in the field of view of the slit monitor camera. There are two adjustment knobs on each mirror to align the image. If the image is centered but rotated, work the adjustments of the two mirrors against each other to straighten up the slit in the image while keeping the image centered.

2) Slit illuminator brightness. There is a potentiometer knob or set-screw located on the outside of the foreoptics box to adjust the brightness of the green LED. At Site 300, the LED brightness is adjusted at the video switch box on the analyzer table.

3) Focus image of slit in monitor camera. There is a ring around the lens housing in the slit monitor box that adjusts the focus of the image for the slit monitor.

4) Slit rotation. Sometimes the slit is rotated with respect to the fringes produced by the analyzer table. There is a set of screws on the slit mounting assembly to correct this. This generally takes a few attempts to set properly. There are two warnings to consider before doing this: 1) check to make sure there is not an alignment problem on the analyzer table, 2) you might rotate the fids off the input slot of the streak tube (see step 5 of Section 5.3.1).

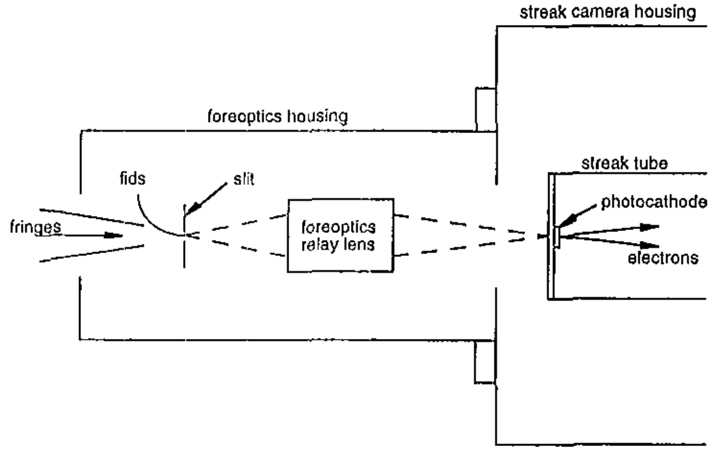


5) Fid intensity on the film. Fibers carry the fid pulses from the laser diode to the slits inside the foreoptics box. The intensity of the fids on the film are adjusted with fiber attenuators mounted on the analyzer table in front of the streak cameras.

6) Focus the image of the fids on the film. There is a ring around the foreoptic relay lens to adjust the focus of the slit image to the photocathode. The easiest way to view this adjustment is by looking at polaroid film. When satisfied, expose a piece of hard film to ensure that the images of the fids have sharp well-defined edges. (Note: Another cause of poor fid focus is if the MCP has not been properly seated against the back of the streak tube. If you have never done this, ask someone who has.)

7) Shutter. There are no adjustments to make on the shutter. Sometimes the shutter fails, however, or other times you may wish to view the fringes directly on the streak tube photocathode. In these cases, the shutter may be removed. Unplug the connector at the housing of the foreoptics box and unscrew the entire shutter housing.

The relay lenses in the foreoptics transport light from the slit to an image plane at the front of the streak camera tube. This image at the output of the foreoptics contains light from the fids and from the fringes themselves. This light must land upon the photocathode at the front of the streak tube.



The photocathode input slots have different widths in the different types of camera tubes:

Streak camera	Input slot width (mm)
HHP	3
EGG red	7
BN black	1

When mounting a foreoptics box to a streak camera, there are several adjustments that are important to make:

1) Focus: There is a ring around the foreoptics lenses to adjust the focus. This must be adjusted so that the slit makes a sharp image at the input slot of the streak tube. Use polaroid film to get this as close as possible, and then verify by exposing some hard film.

2) Aperture: There is a second ring around the foreoptic relay lens that adjusts the aperture. This aperture needs to be fully open. If the image at the film is cut off around the edges, then the aperture is not open all the way.

3) Magnification: The relay lens inside the foreoptics box may be slid forward and backward to adjust the magnification. Generally, we set the overall magnification of the foreoptic/streak-camera system to be approximately 2. The fibers that carry the fids to the slit are spaced 12 mm apart. Measure the spacing between the fids on polaroid film and slide the foreoptic relay lens until the fids on the film are approximately 24 mm apart. If one or both fids disappears while increasing the magnification, the fids have moved off the streak tube input slot. Decrease the magnification until both fids reappear.

These next two steps need to be performed together.

4) Slit alignment: The slit must be located laterally so that its image lands near the center of the input photocathode. Use the fids for this adjustment.

Left/right--Move the slit, say, to the left until the fids disappear (looking at the tube output), and then move the slit to the right until they disappear again. Return the slit to the center of these two limits.

Up/down--Expose a polaroid and count the number of dots across the top and the bottom. If the film shows more fid dots on the top than on the bottom, or vice versa, then the slit height is not set properly.

5) Slit rotation: The slit must be rotated so that the image of the fids and fringes are parallel to the input slot. Use the fids for this adjustment. When the left/right adjustment in step 4 is performed, note whether top and bottom fid disappear at the same time. If not, then the slit rotation needs to be adjusted.

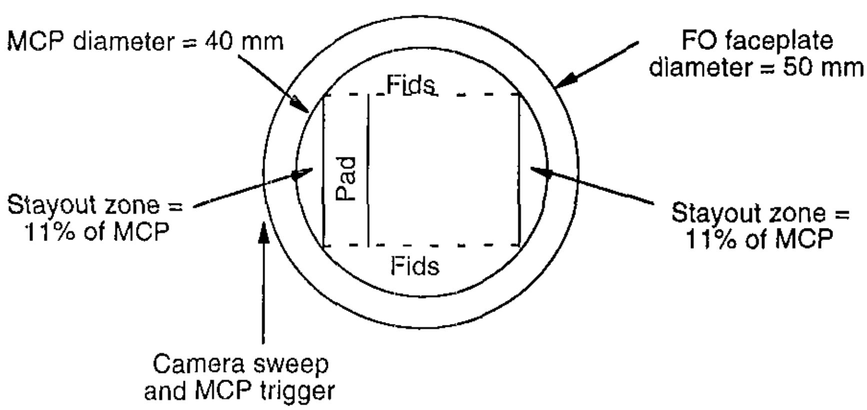
Note: Setting the slit rotation properly to the input slot is particularly important for the Bechtel black cameras because the input slot is so narrow. If the fringes are not aligned properly with the slit once the black cameras are mounted on the analyzer table, it is necessary to tilt the entire camera with shims rather than tilting only the foreoptics slit as mentioned in step 4 of Section 5.2.2.

The output end of the streak tube is a round fiber optic (FO) faceplate. The inside surface of the faceplate is coated with the phosphor. Because the phosphor does not produce enough light to expose our film, we also install a micro-channel plate (MCP) intensifier onto the back of the tube. All of our cameras have MCPs which are 40 mm in diameter and the film is pressed against the MCP during the shot. The film is exposed only within the MCP area. The useful region that our data fills on the film is defined by the top and bottom fids and is, therefore, rectangular. This is the classic problem of a square peg in a round hole. We are not able to use all of the MCP area for our data. Another consequence of this geometry is that the times to trigger the cameras is not at the beginning of the usable record.

The following diagrams show the tube/MCP output geometry for the three different types of streak cameras. Within each type, every camera is a little different, so the actual times will be somewhat different from given below. Set up the camera sweeps and triggers as given below. Count the total number of fid dots to get the usable record length. You may end up choosing a different sweep length. Also count the number of dots to the report and adjust the trigger time to get the desired pad.

HHPs

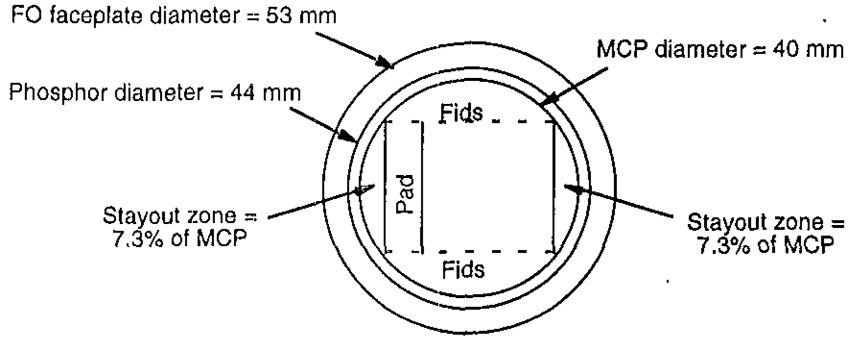
When you set the sweep time S for the HHPs, you are setting the time to sweep across the tube diameter which is 50 mm. The MCP is only 40 mm diameter, so the time to sweep across the MCP is 0.80 x S. Furthermore, there is some dead space on each end of the usable area which accounts for 11% of the MCP diameter on each end. The result is that the time of the usable record (fid length) is 0.624 x S and you will need to trigger the camera 0.188 x S earlier than you want the fids to start.



Trigger the camera at time = T, and set the sweep length = S.
Fid start = T + 0.188 x S
Fid end = T + 0.812 x S
Fid length = 0.624 x S

EGG Reds

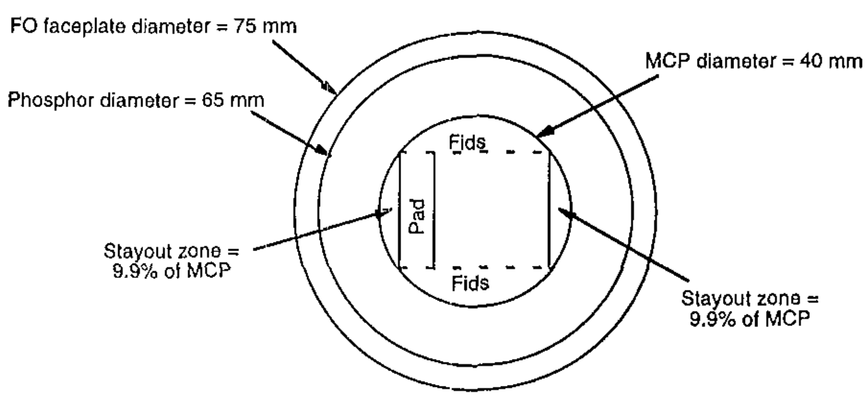
The sweep boards for the Red cameras have been adjusted so that the usable record length (fid length) is approximately equal to the time set S. This means that the trigger times depend upon the sweep length chosen. The trigger times and fid lengths were compiled for the recent Oboe series at U1a. The average fid length was 0.93 x S. The average times from the trigger to the start of the fids is summarized in the following table. There is considerable camera-to-camera variation, so some adjustments in the trigger times will be needed each time the sweep setting is changed.



Sweep Setting (μs)	Trigger to Fid Start (μs)
6	5.1
12	6.6
20	8.8
30	11.4

Bechtel Blacks

At this writing, only two of the black streak cameras have been used at U1a. Both were used at a sweep setting S of 3 μs. The fid length was approximately 2.95 μs = 0.98 x S for both cameras. The triggers were approximately 4 μs before the fid start.



HHPs: There are two types of HHPs—most of the cameras have a single sweep rate while some of the cameras have the capacity to change to a second sweep rate halfway through the sweep (dual sweeps). The sweep times are given in microseconds:

Single Speed Cameras				Dual Speed Cameras		
Slow Range	Medium Range	Fast Range		Slow Range	Medium Range	Fast Range
15	3	0.3		11	2	0.3
20	4	0.4		12	3	0.4
25	5	0.5		14	4	0.5
30	6	0.8		16	5	0.6
40	7	1.0		18	6	0.8
50	8	1.3		20	7	1.0
60	9	1.6		25	8	1.2
70	10	1.8		30	9	1.4
80		2.0		35	10	1.6
				40	15	1.8
				45	20	
				50	25	
				60	30	
				70	35	
				80	40	

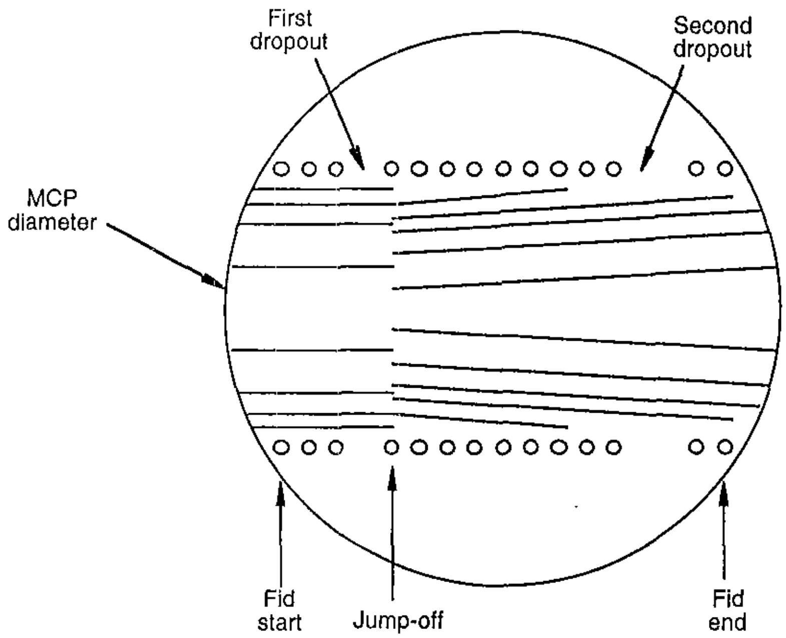
Notes:
1) For sweeps shorter than 0.5 μs, the fid pulse width may be longer than desired.
2) For the dual sweep cameras, the two sweep times must be within the same range.
3) The useable data lengths are 0.624 times the sweep lengths given above.

EGG Reds and Bechtel Blacks: These cameras have a smaller selection of sweep times available. The sweep times are given in microseconds:

Reds	Blacks
3	3
6	
12	10
20	20
30	30

Notes:
1) At U1a, the fids have 7 ns FWHM, which are narrow enough for 3 μs sweeps.
2) An internal pot will allow the Blacks to be set to 2 μs sweeps, but this needs to be done in the streak camera lab so various voltages may be monitored.
3) The useable data lengths are 0.93 (Red) and 0.98 (Black) times the sweep lengths given above.

During a shot, the streak camera produces an image on the film that has some of the features shown below. Time increases from left to right. The film record is contained within the area defined by the MCP. For our standard streak records, the image is symmetric about a horizontal line in the middle of the record.



Fids: The row of dots across the top and bottom of the data are called the fids. These are produced by the FTG (section 7.2.1) and are delivered to the slit in the foreoptics via fibers. The fibers are 12 mm apart at the slit and the slit itself is 11 mm tall.

Dropouts: These are the missing fids and occur when the experimenter sets the report times (section 5.6). There is one missing dot, called the single dropout, at the first report and two missing dots, called the double dropout, at the second report. There can also be a third dropout although this is rarely used.

Jump-off: During measurements of surface velocity, the initial shock often changes the velocity from zero to some value so quickly that the analyzer table cannot follow it. The fringes showing zero velocity, coming in from the left, suddenly disappear and new ones form at different distances from the center. This abrupt change in fringe position is called jump-off. The time at which jump-off occurs is often of great interest to the designer. This sketch shows data with the jump-off arriving approximately one dot after the first report and with slowly increasing velocity during the record.

Pad: The pad is the amount of time from the start of the fids to the expected time of jump-off. This time is made long enough to assure that the jump-off will be observed even if the jump-off occurs earlier than the designer expects. One cannot tell how long the pad is by looking at a streak record.

There are no strict rules regarding streak camera coverage on experiments. Some of these suggestions are based upon personal experience and others are from comments of others.

Sweep length and Pad: These are usually determined by the experiment designer. He will want to follow the surface for a given distance and will usually have an expected velocity. From these, you can calculate the length of time to follow the surface. The length of the pad at the beginning of the coverage is usually determined by the designer's uncertainty in when the measurement should start. Avoid the temptation to make the pad too short. For long sweeps (> 20 μs), I might make the pad10% to 20% of the fid length and, for short sweeps (< 5 μs), I might make the pad 30% to 50% of the fid length. Add the pad time and the time to follow the surface. Divide this time by 0.624 (HHP) or 0.92 (Red) or 0.96 (Blacks) to estimate the sweep time to enter on the streak camera. Count the number of fid dots on the film record to assure that you have the desired coverage.

Fid interval: The fid interval (time per dot) is usually established by the number of dots to have on the film record. If there are too few dots, the analysis code must interpolate over a relatively long time between dots, which increases the timing uncertainty. If there are too many dots, it becomes difficult to count them during camera set up. I like to have at least 10 dots on a record, knowing that the reports will remove at least one or two of them. I have set up records with nearly 50 dots, but 30 or so dots is a more comfortable maximum. Keep in mind that a single fid generator provides the fids to all cameras on a single table. The sweep times of the cameras on a given table must stay within a limited range to maintain the desired range of dots on all the records. For this reason, we may arrange the coverage to have one analyzer table with slow sweeps and a second table with fast sweeps. Each table then would have its own fid generator set for the appropriate fid interval.

Reports: Where to put reports on a film record is sometimes a matter of personal preference, and sometimes a matter of necessity. On each analyzer table, a single fid generator drives all the cameras, so the report times must be chosen to accommodate the data expected for all of the cameras on that table. If all the cameras expect identical jump-off times, I will often set the first report to coincide with that time. This way, I can tell immediately upon seeing the film whether the experiment was early or late and by how much. If the cameras on a given table expect different jump-off times, then I will often set the first report to coincide with the latest jump-off time. I will then stagger the camera trigger times to give the same number of dots to expected jump-off for all the film records (in other words, keep the pad the same for all cameras regardless of where the report is). A second report is required if the expected stagger in events is so great that a single report would not be observed on all cameras. Sometimes a second report is added near the end of the camera sweeps merely to minimize confusion during set up.

Streak camera gains: The ball-roll surface preparation technique appears to do a good job of minimizing variations in light intensity during the measurements. The streak camera gains are usually set by looking at polaroid film prior to the shot. If the fringes on the polaroid are just starting to saturate, then the hard film will usually turn out well. Fortunately, the dynamic range of hard film is much greater than that of polaroid, so the polaroid may be quite saturated during set up and the hard film will still be okay. If the surface has poor reflectivity and the light return is very weak, then increased streak camera gains will result in a large amount of MCP noise before the desired fringe intensity is obtained. In these cases, I generally set the camera gain for a tolerable amount of MCP noise and hope the fringe brightness does not decrease during the measurement.

Fid intensity: The fid intensity should be set after the streak camera gains have been set according to the desired fringe intensity. Avoid the temptation to make the fids too bright. If the fid brightness saturates the polaroid, I would suggest decreasing the fid intensity. The fids can appear to be quite weak on the polaroid and still be sufficiently bright on the hard film. If time permits, expose a set of hard film prior to the shot and examine the fids with an eyeloop. The edges of the fids on hard film should appear sharp and well-defined. (If the fids are obviously not saturated and still have soft-looking edges, you may have a focus problem. See Sections 5.2.2 and 5.3.1 on foreoptics adjustments.) One problem that sometimes occurs is the case where a large amount of light is received from the experiment. To obtain the desired fringe intensity, the streak camera gains may be turned down so far that the fids are now too weak even when turned up all the way. In this case, tape a Wratten filter of, say ND 1.0, to the front of the foreoptics box and turn the camera gain back up.

Section 6

Film and Filters

We use both polaroid film and hard film with the Manybeam system. The hard film is expensive and takes a long time to develop, so we use polaroid film when setting up the analyzer table at the beginning of the day or in preparation for a shot. Generally, we use polaroid film to set streak camera trigger times, camera gains, fid intensity, and focus. At HEAF and Site 300, we usually use polaroid for the final dry run, also, but at U1a we always use hard film for the final dry run. All facilities use hard film for the shot itself because hard film has much more dynamic range and spatial resolution. The film back holders on all of our streak cameras are compatible with both polaroid film cassettes and hard film cassettes.

We use polaroid film with 3000 ASA to closely match the speed of the hard film. This allows us to set streak camera gains using the polaroid with assurance that the exposure on the hard film will be correct. The HHPs use a different brand of polaroid than the EGG Reds or the Bechtel Blacks:

HHPs	Polaroid T-667 High Speed Professional Coaterless B&W Instant Pack Film ISO 3000/36° Processing 30 sec. at 75°F 3.25" x 4.25" 10 prints per pack, 2 packs per box Web site: www.polaroid.com/service/filmdatasheets/3_4/667fds.pdf
Reds/Blacks	Polaroid T-57 High Speed Black & White Peel Apart Film ISO 3000/36° Processing 15 sec. at 75°F 4" x 5" 20 prints per box Web site: www.polaroid.com/products/instant_cameras/peelapart/4x5/index.html#57

All facilities use hard film for the shots themselves because hard film has much more dynamic range and spatial resolution than polaroid. At U1a, we also use hard film for the final dry run and some of the mandatory dry runs.

We use Kodak T-MAX 3200 ASA hard film in 4" x 5" sheets. This is not off-the-shelf film and must be special ordered. The minimum order is 5000 sheets and is fairly expensive. In June 1997, Rex Avara placed an order for 5000 sheets for around $6K, and it took 3 months to receive the order. Attention must be paid to the stock supplies to assure that a new order is placed with sufficient lead time. All of our streak cameras use this same type of hard film, so the cost of the film order is sometimes shared among the various facilities. This film has a shelf life of about 3 years at room temperature, so we keep the film in a freezer to extend its shelf life. It is not unusual for the stock of film to go beyond its stated expiration date before we use it up.

Kodak Professional Film
T-MAX P3200
Black & White
4" x 5"
Web site:
www.kodak.com/global/en/professional/support/techPubs/f32/f32Contents.shtml

The following table gives guidelines regarding film developing times (in minutes) vs. chemical temperature (in degrees F) for T-MAX P3200:

Temp (°F)	Time (min)
68	11
70	9.5
72	8.5
75	7.5
80	6.0
85	5.5

Shot film is sometimes pushed one f/stop or even two f/stops depending upon whether there is a fear that the fringe intensities may become weaker once the experiment starts. With the ball-roll surface preparation technique, we rarely feel the need to push the film developing.

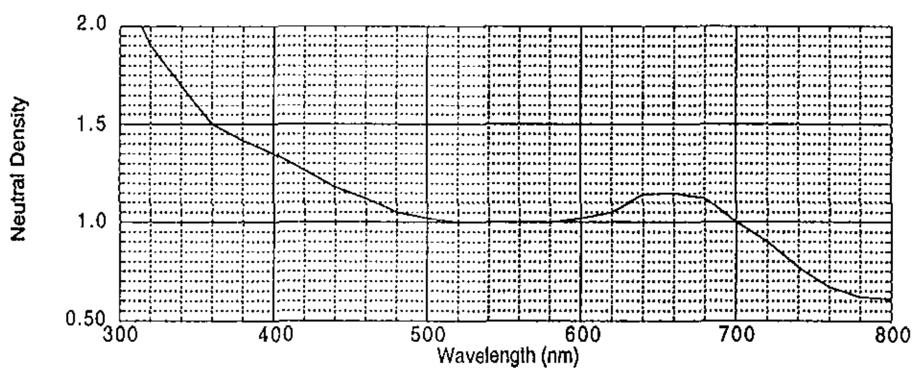
We use neutral density (ND) filters in various locations on the analyzer tables. Probably the most important application is to reduce the fringe brightness into a streak camera on those rare occasions when we have too much light returning from a probe. This sometimes happens when we have the probe mounted closer to the experiment than usual or if the experiment surface is highly reflective. We usually discover that this is a problem only hours before a shot when it is time to set the final streak camera gains after the experiment has been installed. During high-power dry runs, we will find that turning the streak camera gain all the way down to zero still results in fringes that are too bright. Or we may be able to turn the gain down enough to obtain the proper fringe brightness, only to discover that the gain is so low that we cannot increase the fid brightness enough. In these cases, we merely tape a piece of ND filter over the hole at the entrance to the foreoptics box and then increase the MCP gain to a more useable level.

We use Kodak Wratten ND filters because they are very thin and do not change the focal plane position at the slit. (See Section 8.2, Example 3 for a discussion on fore-shortening of focal planes.) The ND values of transmission are based upon a logarithmic scale:

$$ND = -\log(Transmission)$$

ND value	%Transmission	Attenuation	Increase in f/stop
0.1	80	1.25	0.33
0.2	63	1.5	0.67
0.3	50	2	1.00
0.4	40	2.5	1.33
0.5	32	3	1.67
0.6	25	4	2.00
0.7	20	5	2.33
0.8	16	6	2.67
0.9	13	8	3.00
1.0	10	10	3.33
2.0	1	100	6.67
3.0	0.1	1,000	10.00
4.0	0.01	10,000	13.33

We tend to think of ND filters as having constant transmission at all visible wavelengths, but their transmissions do vary slightly. The following is an example for an ND=1.0.



All of our analyzer tables have a bandpass filter located between the pick-up lens and the cylinder lens (as shown in the sketch in Section 4.1). These filters are made by Barr Associates, Inc. and have a bandpass width of approximately 1 nm centered around 532 nm. They have a peak transmittance of approximately 70% and should be mounted on the table with the silver side facing upstream toward the pick-up lens.

It is very important to make sure that the bandpass filter is mounted on the table during integrated dry runs and during the shot:

1) For laser safety: On some shots, it is possible that other lasers with different wavelengths, such as the ruby laser, are being used on the experiment. Light from those lasers could enter our fibers and be propagated to the analyzer table. Our laser safety goggles block our own laser but not others. The bandpass filter prevents light from other lasers from becoming a laser safety issue in the analyzer room.

2) For data quality: On some shots, very bright broadband light sources, such as explosively-driven candles, are used for illumination for rotating mirror cameras. Also, the high-explosive driving the shot itself produces a bright flash of broadband light. If there were no bandpass filter on the analyzer table, this broadband light would pass through the analyzer table and be recorded by the streak cameras. This light does not produce fringes, but instead fills in the spaces between the fringes making it more difficult to read the data--especially in the case of weak fringes. In addition to these broadband sources, light from other lasers can also propagate to the streak cameras. For example, when the ruby laser fires, it can produce narrow vertical lines on the streak camera data record.

Notes:

1) Be sure the filter is not tilted at a large angle with respect to the beam. This will greatly attenuate the light to the streak cameras.

2) On the other hand, be sure the filter is not exactly perpendicular to the beam, either. The filter has a reflectivity of approximately 30% and can be a source of conjugate crosstalk, that is, light from fiber 1 could end up on camera 5, etc.

3) During table set-ups and morning checks, look at the reflection from the bandpass filter and make sure it is just outside the exit aperture of the shaper.

4) Barr Associates has a very informative website (www.barrassociates.com) that gives definitions for various terms used to describe bandpass filters.

Section 7

Data Analysis

Section 7.1

Data Analysis
Velocity

The formula to calculate velocity v_j from a data record is:

$$v_j = f_c\left(\frac{D_j^2 - D_1^2}{D_2^2 - D_1^2} + n_j\right)$$

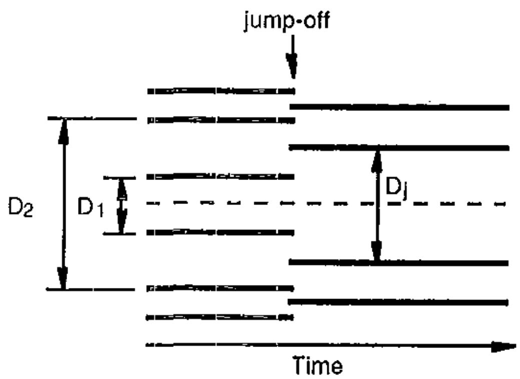
where the fringe constant f_c is

$$f_c = \frac{c\lambda}{4\left[H + T\left(n - \lambda\frac{dn}{d\lambda}\right)\right]}$$

$$f_c[mm/\mu s] = \frac{39.88}{H[mm] + 1.485 * T[mm]}$$

where the variables in the fringe constant are defined in section 4.6.4, and

D_j = fringe diameters on the data record as shown below
n_j = integer number of fringe jumps



Notes:

1) The quantity $\frac{D_j^2}{D_{j+1}^2 - D_j^2}$ is called the fractional order. The expression for velocity calculates the change in fractional order from the pre-jump velocity (zero) to the post-jump velocity.

2) The quantity $\Delta^2 = D_{j+1}^2 - D_j^2$ is a constant for a given cavity. This is how we identify which fringes in a data record are from the primary cavity and which are from the referee.

A common expression for the velocity resolution Δv is a function of the fringe constant f_c and the etalon finesse F:

$$\Delta v = \frac{f_c}{2F}$$

Many of our etalons have F = 20, or so.

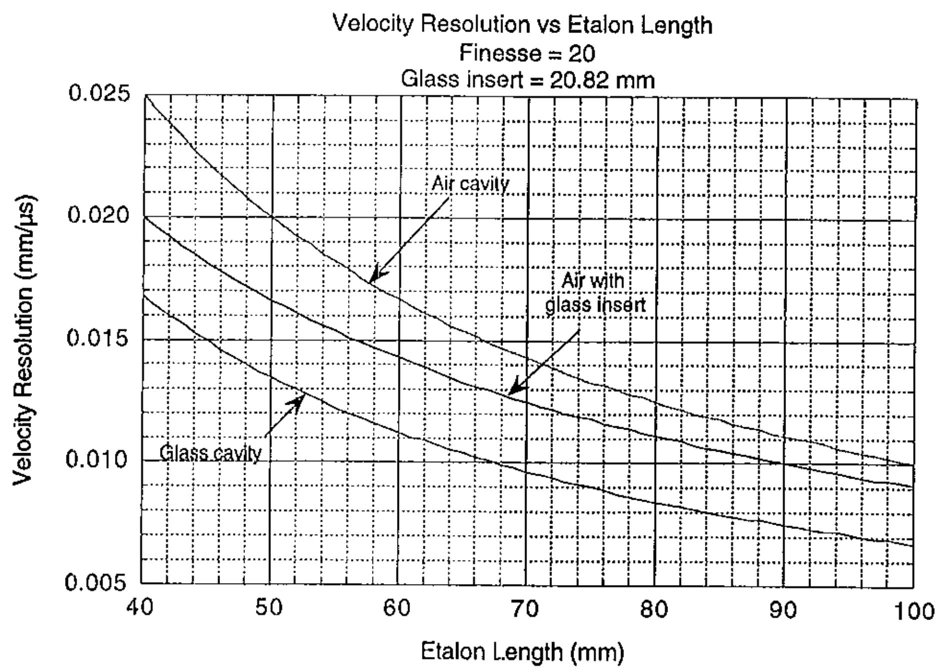
Recall that (from section 7.1.1)

$$f_c[mm/\mu s] = \frac{39.88}{H[mm] + 1.485 * T[mm]}$$

so

$$\Delta v\ (mm/\mu s) \doteq \frac{1}{H(mm) + 1.485 * T(mm)}$$

In the following graph, the "Air with glass insert" cavity has an air-only primary cavity with length H_0 and an air/glass referee cavity with glass length T = 20.82 mm and air length H = (H_0 - 20.82) mm. In this case, the x-axis refers to the length of the air-only primary H_0:



Notes:
1) The expression for velocity resolution does not include the effects of non-perfect optics or streak camera spatial resolution or distortion.
2) For high velocity resolution, the best cavity to use is a long all-glass etalon.

Having a referee cavity on the analyzer table does not yield only a single velocity solution to the data, but rather allows an infinite number of exact or near solutions at higher velocities as well. With only a single cavity on the analyzer table, allowed solutions to the data differ by only the fringe constant of the etalon; this difference can sometimes be smaller than the uncertainty in the expected velocity. The advantage of having a referee cavity is that the next velocity allowed by the combination of etalons can be made much higher than in the case of a single etalon alone. In fact, the next velocity allowed by two etalons can be made higher than any velocity physically possible in the experiment.

Assume the primary cavity has fringe constant f_p and the referee cavity has fringe constant f_r. If the 1st velocity allowed by this combination is v1, then the next velocity allowed is v2.

$$v1 = f_p\,(fo_p + n1) = f_r\,(fo_r + m1)$$
$$v2 = f_p\,(fo_p + n2) = f_r\,(fo_r + m2)$$

where fo_p and fo_r are the change in fractional orders of the primary and referee cavities, respectively, that resulted from a given measurement. The number of fringe jumps for the primary cavity from v1 to v2 is n2 - n1 = Δn, and the number of fringe jumps for the referee cavity is m2 - m1 = Δm. Subtracting the top equation from the bottom equation and eliminating fo_p and fo_r yields the following relationship:

$$\frac{\Delta n}{\Delta m} = \frac{f_r}{f_p}$$

The ratio of fringe jumps to get to the 2nd allowed velocity is just the ratio of the fringe constants. If the ratio of fringe constants cannot be expressed exactly as the ratio of integers, then the match between the two etalons will not be exact for higher allowed velocities.

Tips on choosing a pair of etalons for a given experiment:

- choose etalons whose fringe constants are not near the ratio of small integers,
- choose shorter etalons if the uncertainty in the velocity is large,
- choose shorter etalons for higher velocities to reduce the number of fringe jumps

The next page gives two examples of combining etalons with different fringe constants. In these examples, the ratio of the fringe constants may be expressed exactly as the ratio of two integers. Also, f_p and f_r are the fringe constants (in mm/μs) of the primary and referee cavities, and Δv is the velocity difference (in mm/μs) between the 1st and 2nd velocity solutions.

Example 1:

The ratio f_r/f_p is held constant at 1.50 and the lengths of the two etalons are varied. Recall that a shorter etalon has a larger fringe constant, so for the three examples given here, the cavity lengths are decreasing from top to bottom. The number of fringe jumps for each cavity is varied until the 2nd velocity match is found. The ratio of the fringe constants f_r/f_p is 3:2, so the ratio of fringe jumps $\Delta n/\Delta m$ is 3:2.

f_r/f_p	f_p	f_r	Δn	Δm	Δv
1.50	0.400	0.600	3	2	1.2
1.50	0.500	0.750	3	2	1.5
1.50	0.600	0.900	3	2	1.8

Note: The 2nd allowed velocity increases with shorter etalons.

Example 2:

The fringe constant f_p of the primary cavity is 0.50 mm/μs, and the fringe constant f_r of the referee cavity varies from 0.55 mm/μs to 1.00 mm/μs in steps of 0.025 mm/μs. The first column shows the ratio f_r/f_p. For each case, the number of fringe jumps for each cavity is varied until a match is found for the 2nd allowed velocity. Note the wide range of differences between 1st and 2nd allowed velocities.

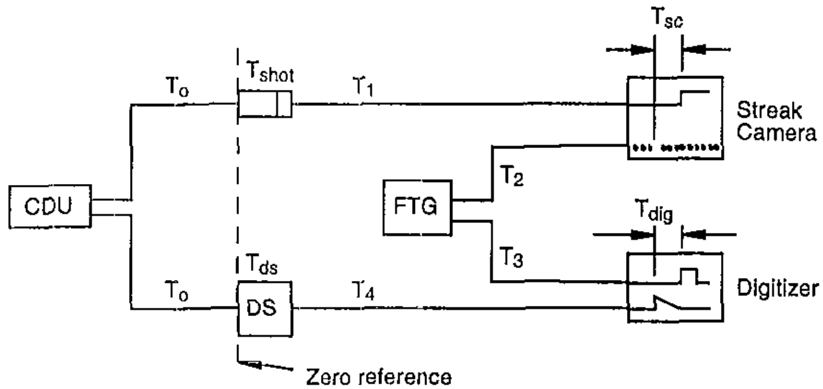
f_r/f_p	f_p	f_r	Δn	Δm	Δv
2.00	0.500	1.000	2	1	1.0
1.95	0.500	0.975	39	20	19.5
1.90	0.500	0.950	19	10	9.5
1.85	0.500	0.925	37	20	18.5
1.80	0.500	0.900	9	5	4.5
1.75	0.500	0.875	35	20	17.5
1.70	0.500	0.850	17	10	8.5
1.65	0.500	0.825	33	20	16.5
1.60	0.500	0.800	8	5	4.0
1.55	0.500	0.775	31	20	15.5
1.50	0.500	0.750	3	2	1.5
1.45	0.500	0.725	29	20	14.5
1.40	0.500	0.700	7	5	3.5
1.35	0.500	0.675	27	20	13.5
1.30	0.500	0.650	13	10	6.5
1.25	0.500	0.625	25	20	12.5
1.20	0.500	0.600	12	10	6.0
1.15	0.500	0.575	23	20	11.5
1.10	0.500	0.550	11	10	5.5

Note: When Δn and Δm can be expressed as small integers, then Δv is small also.

Section 7.2

Data Analysis
Timing

All of our experiments, whether at HEAF, Site 300, or U1a, are set up generally in the same manner.



CDU: The capacitive discharge unit (CDU) fires the shot with a high-voltage, high-current pulse. A small fraction of the pulse is also tapped off to a "det shaper" (DS). The cable lengths T_o from the CDU to the shot and from the CDU to the DS are usually kept the same to establish a "zero reference" time.

DS: The det shaper (DS) splits the signal from the CDU into many smaller signals which are then sent to timing digitizers for each diagnostic. T_4 is the cable delay from the DS to the digitizers.

T_{shot}: Once the pulse reaches the shot, the bridgewire bursts which lights the high-explosive (HE). It sometimes takes quite a bit of time before the shock breaks out at the surface of the experiment. The designers are very interested in this time (jump-off time).

T_1: This is the time for the signal to travel from the shot to the streak camera. T_1 includes the air path from the shot surface to the probe, the length of fiber, the air path on the analyzer table (32 ns), and the fill time of the etalon.

FTG: The fiducial timing generator (FTG) has two outputs which are timed very precisely with respect to each other. One output sends out one to three pulses, called reports, that are sent to the digitizer via a cable with delay T_3. The times of these reports may be adjusted by the experimenter according to the details of the shot. (Setting report times is discussed in section 5.6.) The second output of the FTG sends out a burst of pulses, called fids, that are sent to the streak cameras. The time interval between fids may be adjusted by the experimenter. When the first output sends out the first report, the second output skips a fid (this is called a dropout). When the first output sends out the second report, the second output skips two fids, and so on.

T_2: This is the total time to travel from the FTG to the streak camera. The burst output of the FTG is a series of electrical signals. These are converted to a burst of optical signals by a laser diode (800 nm) before being sent to the streak cameras. Usually, a single laser diode feeds all the streak cameras on an analyzer table via an optical splitter.

For each experiment, it is very important to precisely correlate times in the streak camera record with events in the shot. Unfortunately, the data on the streak cameras cannot be precisely timed to the shot without knowing exactly when the CDU fires. This is why we send a small fraction of the CDU pulse to a digitizer, and then use the FTG to correlate times in the digitizer record with times in the streak camera record. To do this, we must take into account the transit times of the various components in the setup shown in section 7.2.1. Notice that the setup builds a complete loop of different signals and different data records. To time the streak camera record with respect to the shot, we "close the loop." The following discussion refers to the figure in section 7.2.1.

To close the loop, start at the zero reference point just before the shot and follow the path through the various components and delays back to the zero reference at the DS. When you go with the direction of signal, assign a "+" sign to those delays, and when you go opposite the direction of the signal, assign a "-" sign to those delays. Assign plus or minus signs to the streak camera read T_{sc} and digitizer read T_{dig} in the same way. The algebraic sum of all the delays from the zero reference back to the zero reference should add up to zero:

$$T_{shot} + T_1 - T_{sc} - T_2 + T_3 - T_{dig} - T_4 - T_{ds} = 0$$

Note that T_{dig} has a minus sign because the path went against the direction of time from the FTG signal to the DS signal on the digitizer. The sign of T_{sc} is minus for the same reason; if the shot jump-off were earlier than the FTG signal, then the sign of T_{sc} would have been positive. Solve for T_{shot}:

$$T_{shot} = T_{dig} \pm T_{sc} + (-T_1 + T_2 - T_3 + T_4 + T_{ds})$$

Before the shot, determine all of the delay times in parentheses. After the shot, measure T_{dig} and T_{sc} to determine T_{shot}.

Notes:

1) To determine when the reports occur in CDU time, set $T_{sc} = 0$. You will need to have the Control system provide a trigger so that a CDU pulse is recorded on the digitizer.

2) Sometimes it is possible to adjust the cable delays to make the algebraic sum in parentheses equal to zero, so that

$$T_{shot} = T_{dig} \pm T_{sc}.$$

In this case, we say that "the system is zoeroed." Remember, the sign of T_{sc} is positive if jump-off on the streak record is after the report, negative if before.

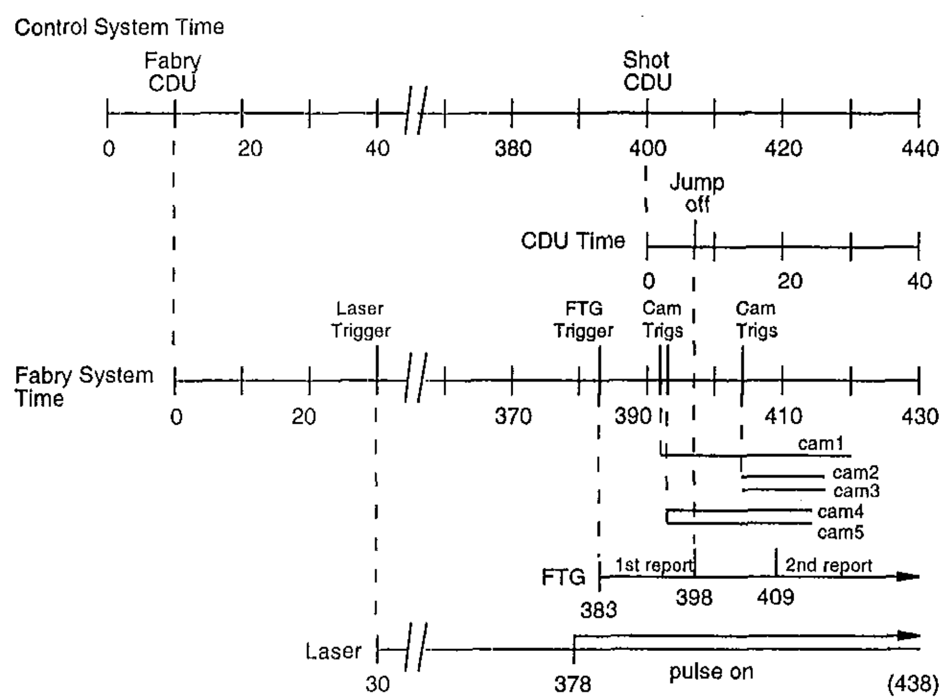
There are three sets of reference times in every shot. The Control System has its reference time, each diagnostic system has its own reference time, and the designer often discusses events with respect to some time within the shot itself.

Control System Time: In every shot, the bunker Control System starts the chain of events that leads up to firing the shot. The Control system fires a series of CDUs that send triggers or timing information for the entire suite of diagnostics. One of these CDUs fires the shot itself and is called the "shot CDU". Zero time for the Control system is often called coincidence time.

Fabry System Time: The Control system needs to know when to fire a CDU for the trigger for each diagnostic. It is the responsibility of the lead experimenter for each diagnostic to give that information to the Control system. For the Fabry system, the laser needs the longest time between triggering and firing--generally between 250 μs and 350 μs. Be sure to check with the laser operator at the facility where you are setting up to take data. The Fabry system receives its system trigger from the Control system, and then the Fabry system generates its own set of triggers for the laser, FTG, and streak cameras. (At B851, the option exists to trigger the streak cameras directly from the shot CDU.)

CDU Time: The experiment designer performs his calculations starting at some point in the high-explosive (HE) geometry. The lead diagnostic person must talk to the designer or to the ramrod to determine how much time to add to refer to the shot CDU.

The following diagram shows a set of timelines from a typical shot:



Time resolution refers to how quickly the Manybeam system can respond to a change in the velocity of an experiment. The initial shock arrival often changes the velocity so fast that the Manybeam system cannot follow the change. The zero-velocity fringes just disappear and re-appear at new locations, hence the term "jump-off". Subsequent shocks may also cause abrupt changes in velocity. Ringing of hard sample materials will create a rapidly-varying velocity time history. The details of these events may be washed out if the diagnostic is not able to respond quickly enough.

There are a number of factors that limit the time response of the Manybeam system. While there are others, the list below provides the dominant factors:

Slit width: The streak camera slit is imaged to the film with a magnification of approximately 2 (see section 5.3.1). This means that the image of a 50 μm wide slit will be 100 μm wide at the film. This image is swept across the film during a measurement and limits the time response of the system. Using the full slit width for this calculation appears to be too pessimistic. With sufficient intensity in the fringes, it is possible to determine the fringe center to within approximately one-fifth the fringe width. The time response T_{slit} (in ns) due to the slit width is given by:

$$T_{slit} = \frac{strkcammag}{5} x(slitwidth) x \frac{sweeptime}{tubediam} x1000$$

where strkcammag = the streak camera system magnification ≈ 2,
slitwidth = the streak camera slit width (e.g. 50 μm = 0.05 mm)
tubediam = diameter of the output end of the streak tube (in mm)
sweeptime = sweep time entered (in μs)

Spatial resolution: The spatial resolution of the streak cameras is dominated by the MCP mounted on the back of the tube. The EGG Red cameras have been measured to have spatial resolutions of approximately 15 line-pairs/mm (lp/mm), while the Bechtel Blacks have approximately 20 lp/mm. Generally, features can be resolved to a line pair. The time response $T_{spatial}$ (in ns) due to the spatial resolution of the streak camera is given by:

$$T_{spatial} = \frac{sweeptime}{tubediam} x \frac{1}{linepair} x1000$$

where sweeptime and tubediam are defined in Slit width above
linepair = spatial resolution (in line-pair/mm)

Fiber modal dispersion: Modal dispersion is discussed in section 2.4 and is equal to approximately 28 ps/m = 0.028 ns/m. The modal dispersion depends upon the total length of fiber L_{fiber} (in m) from the probe at the experiment to the fiber chuck at the analyzer table. The time response T_{modal} (in ns) due to modal dispersion is:

$$T_{modal} = 0.028 x L_{fiber}$$

Etalon fill time: Fringes do not form immediately upon light first entering the etalon, but rather require a certain amount of time for the etalon to fill with sufficient number of bounces. The concept of etalon fill time is discussed in section 4.6.6. That section also provides the equations to calculate fill time and graphs to quickly obtain an estimate. With sufficient light, fringes can be observed on the film record when the etalon is only partially filled, perhaps only 25% or 50%. A worst-case calculation of time response due to etalon fill time would use the 99% fill fraction.

Film digitizing: Before the data is analyzed by software, it is scanned into image files. Usually the data is scanned at R_{scan} =1000 dots per inch (dpi). No information in the data can be determined with less time than a single digitized pixel. Quite often, if there is some noise in the data, several pixels are required to determine that a feature in the data is real. The number of pixels necessary in a given record must be determined on a case-by-case basis. The time response T_{pixel} (in ns) due to the dwell time of a single pixel is:

$$T_{pixel} = \frac{25.4}{R_{scan}} x \frac{sweeptime}{tubediam} x1000$$

where sweeptime and tubediam are defined in Slit width above

R_{scan} = film digitizing density in dpi (usually = 1000, but make sure)

Section 8

Basic Optics

Light travels at different speeds through different materials. The speed is determined by a parameter called the "index of refraction" which is usually labeled by "n". The speed of light in a material v is given by

$$v = \frac{c}{n}$$

where c = speed of light in vacuum.

$$c = 299.8 \text{ mm/ns} = 299800 \text{ mm/}\mu\text{s}$$

Example 1. The index of refraction of air n = 1.0003, so the speed of light in air is almost equal to c. In almost all of our calculations, we use c as the speed of light in air.

Example 2. The index of refraction of many glasses is approximately 1.46 at 532 nm, so the speed of light in glass—including our fibers—is approximately

$$v_{fiber} = \frac{c}{1.46} = 205300 mm/\mu s$$

It is interesting to calculate the time that it takes for light to travel through various lengths of our fiber compared to the same distance in air.

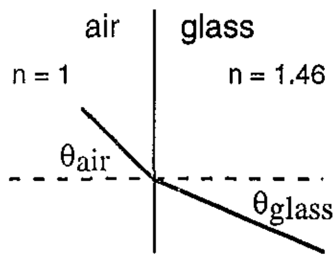
Location	Length (m)	T_{fiber} (ns)	T_{air} (ns)	Diff (ns)
HEAF	10	49	33	16
B851	30	146	100	46
U1a	75	365	250	115

The table in section 9.2 lists the index of refraction for some materials commonly used with the Manybeam system.

When light encounters a material with index n, it not only slows down, but can also change direction--we call this refraction. Refraction is the principle behind all of our lenses. There is a simple equation that relates the change in angle with the change in index of refraction. This equation is called Snell's Law:

$$n_{air}\sin(\theta_{air}) = n_{glass}\sin(\theta_{glass})$$

where n_{air} = index of refraction in air (=1), n_{glass} = index of refraction in glass, θ_{air} is the angle of the incident light with respect to the surface normal, and θ_{glass} is the angle of the light with respect to the surface normal after passing through the surface.



Note: If the light is perpendicular to the surface, the light does not change direction.

Example 1. Total Internal Reflection

When light travels from a material with some index to a material with lower index, it is possible that the light will not refract but will reflect back into the original material. Note in the drawing above that θ_{air} is greater than θ_{glass}. If the light is inside the glass and traveling toward the glass-air interface, it is possible for θ_{glass} to have a value that makes $\theta_{air} = 90°$, that is, the light will just skim along the surface. If $\theta_{air} = 90°$, then $\sin(\theta_{air}) = 1$ and we have (with $n_{air} = 1$):

$$\sin(\theta_c) = \frac{1}{n_{glass}}$$

We call θ_c the critical angle, because with θ_{glass} greater than θ_c, Snell's Law does not apply and all of the light will be reflected back into the glass.

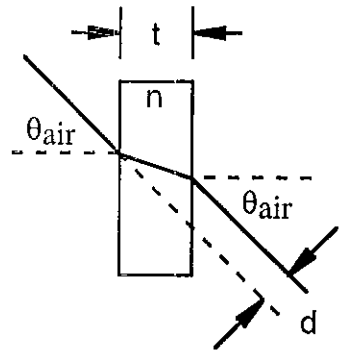
This is the principle behind our optical fibers. We require that the index of the clad be less than the index of the core to achieve total internal reflection. In this case, the critical angle inside the fiber is:

$$\sin(\theta_c) = \frac{n_{clad}}{n_{core}}$$

Example 2. Ray Displacement

When light passes through a plate glass with thickness t and index n, the beam comes out the other side at the same angle that it entered the glass but displaced by an amount d given by:

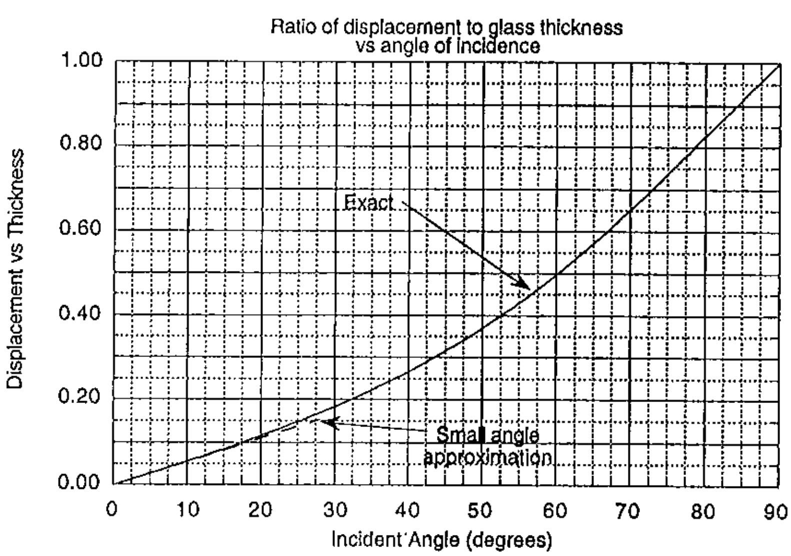
$$d = t * \frac{\sin(\theta_{air} - \theta_{glass})}{\cos(\theta_{glass})}$$



Using small angle approximations, the above expression reduces to:

$$d \approx t * \theta_{air}\left(1 - \frac{1}{n}\right)$$

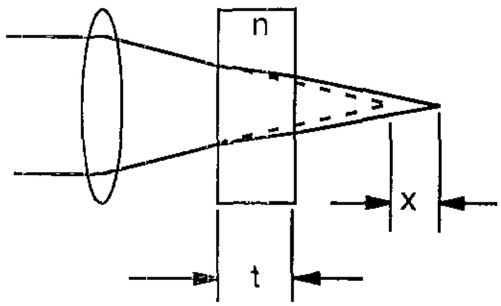
where θ_{air} is expressed in radians. The following graph uses an index of refraction = 1.46:



Note: We use this property to duplex the light from two probes onto a single streak camera.

Example 3. Extending the Focal Plane

If a plate of glass is inserted into a beam of converging rays, the focal plane will be moved away from the lens. The result in Example 2 allows us to calculate how far the focal plane is extended.



If the plate glass has thickness t and index n, the distance x that the focal plane is moved away from the lens is given by:

$$x = t * \frac{\sin(\theta_{air} - \theta_{glass})}{\sin(\theta_{air}) * \cos(\theta_{glass})}$$

Using small angle approximations, this expression reduces to:

$$x \approx t\left(1 - \frac{1}{n}\right)$$

For glass, $n \cong 1.5$, so $x = t/3$, that is, the image is moved away from the lens by one-third the thickness of the glass. This is why we use thin pieces of glass for our duplexing plates; we do not want to change the focal plane of the duplexed fringes any more than we have to.

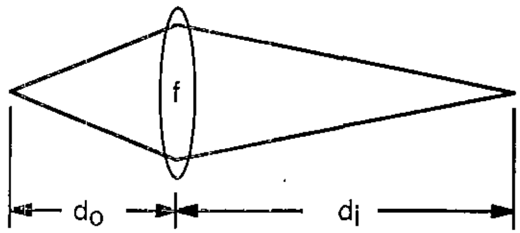
Notes:

1) This expression for x is very similar to the small-angle expression for ray displacements in Example 2.

2) This phenomenon is the reason that objects under water appear to be closer to the surface than they really are.

There is a simple equation to calculate the relationship between the distance d_o from a lens to an object vs the distance d_i from the lens to the image of that object. If the lens has a focal length f, then

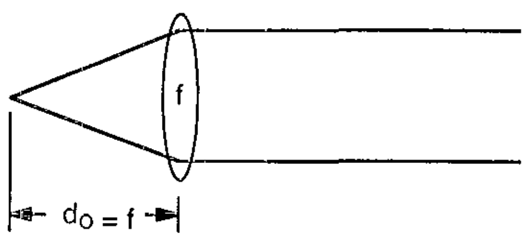
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$



Example: We can use this equation for the relay lenses. The distance from the 5-facet mirror to the relay lens d_o = 37" and the focal length of the relay lenses f = 308 mm = 12.13". This gives a lens-to-slit distance d_i = 18.0", which is approximately half the distance of d_o.

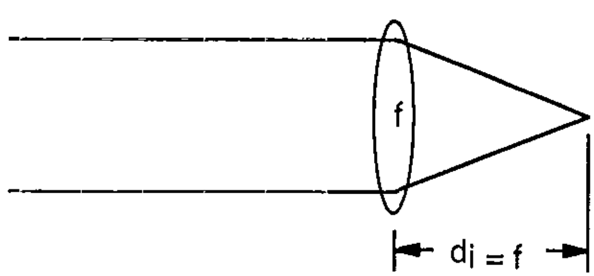
Notes (light travels from left to right in the following):

1. When d_o = f, the image is at infinity, which means that rays from a point on the source end up parallel after going through the lens.



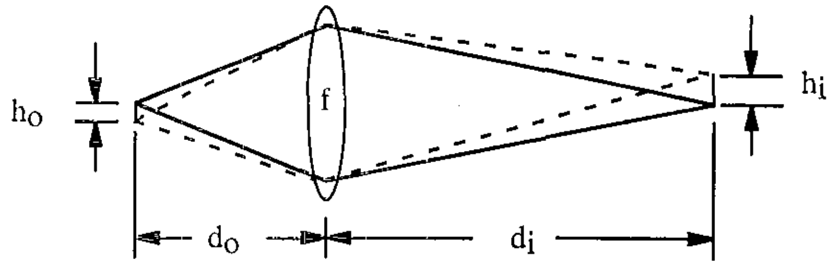
We have this situation with the pick-up lens.

2. When d_i = f, the source is at infinity which means parallel rays are entering the lens and they are focused at the focal plane of the lens.



We have this situation with the spherical lens after the etalon.

Light from different points on an object focus to different points at the image. In general, the size of the image is not always the same size as the object.

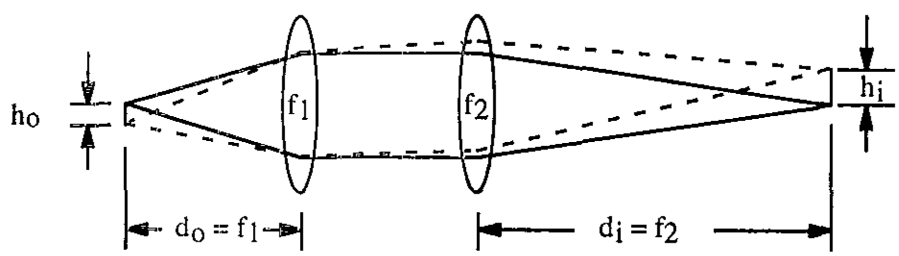


The magnification m is the height h_i of the image divided by the height h_o of the object. This is also equal to the lens-to-image distance d_i divided by the object-to-lens distance d_o.

$$m = \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

Example 1. The distance from the 5-facet mirror to the relay lens is 37" and the distance from the relay lens to the streak camera slit is 18". Therefore, the magnification of the relay lenses is approximately 0.5, so that the size of the fringes on the streak camera slit is approximately one-half the size at the 5-facet mirror.

Example 2. Looking at the pickup lens and the cylinder lens from the side view gives us a special case that allows us to calculate the magnification for a 2-lens system. The image of the shaper is at the focal plane of the pickup lens and the etalon stripe is at the focal plane of the cylinder lens, so that there are parallel rays between the two lenses.



We can calculate the height of the line image at the etalon stripe by rearranging the same equation given above:

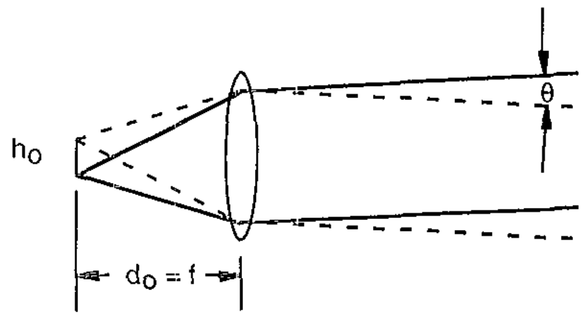
$$h_i = h_o \times \frac{d_i}{d_o}$$

We have $h_o = 0.1$ mm, $d_i = 165$ mm, and $d_o = 1930$ mm, so $h_i = 1.2$ mm. The line focus at the etalon stripe is approximately 1.2 mm tall. Recall that our stripe widths are usually about 0.6 mm wide, so we end up losing some of the light at the stripe.

When $d_o = f$ in the lens equation (Section 8.3), the image is at infinity, which means that rays from a point on the source end up parallel after going through the lens. Rays from a different point on the source end up parallel, too, but at a different angle. If the total height of a source is h_o, then the total range of angles θ from the lens is given by:

$$\theta = \frac{h_o}{d_o}$$

where θ has dimensions of radians.



Example 1.

We have this situation with the pick-up lens. The output of the shaper is at the focal plane of the pick-up lens so we have parallel light exiting the pick-up lens. The focal length of the pickup lens f = 165 mm and the diameter of the fiber core d = 100 μm = 0.1 mm. The shaper produces the football images with 1:1 imaging in the vertical direction (h_v = 0.1 mm) and 4:1 imaging in the horizontal direction (h_h = 0.025 mm). So the ranges of angles of light leaving the pickup lens are

Vertical: $$\theta_v = \frac{0.1\text{ mm}}{165\text{ mm}} = 6 \times 10^{-4}\text{ rad}$$

Horizontal: $$\theta_h = \frac{0.025\text{ mm}}{165\text{ mm}} = 1.5 \times 10^{-4}\text{ rad}$$

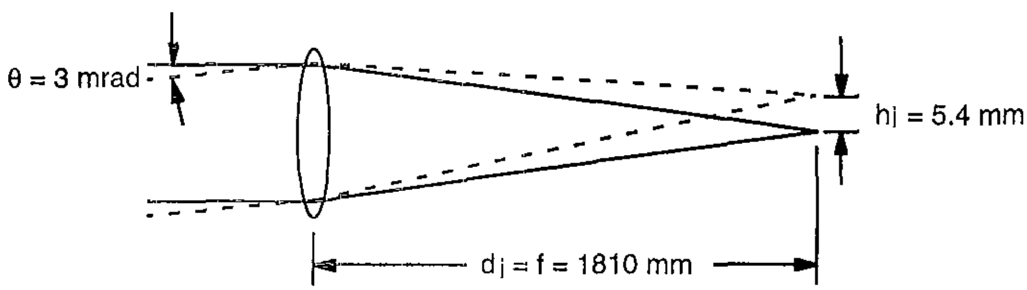
Example 2.

When $d_i = f$ in the lens equation (Section 8.3), the source is at infinity which means parallel rays are entering the lens and they are focused at the focal plane of the lens. Parallel rays entering the lens from different angles are focused at different spots in the image plane.

We have this situation at the spherical lens. The output of the etalon is parallel rays at different angles. They are all focused at the 5-facet mirror but the different angles are focused to different spots on the mirror. We can rearrange the previous equation to calculate where the fringes will occur on the 5-facet mirror.

$$h_i = d_i \times \theta = f \times \theta$$

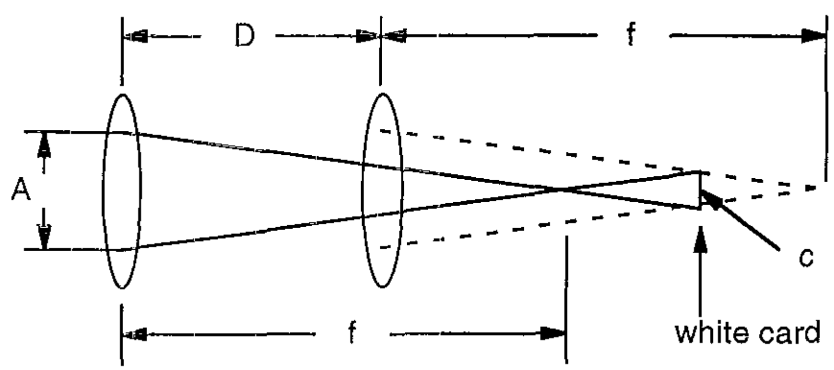
For example, the angle of a j = 1 fringe of a 60-mm-long air-cavity etalon is approximately $\theta = 3$ mrad = 0.003 rad (from the top graph in section 4.6.3). The focal length of the spherical lens f = 1810 mm, so the j = 1 fringe will be approximately h = 5.4 mm above and below the centerline at the 5-facet mirror.



Our eyes are not perfect and cannot tell the difference between an image that is in perfect focus from an image that has a small amount of blur. This means that a point object may be imaged as a small "circle of confusion" before our eyes realize that the point is out of focus. The depth of focus is the total spread of distance on either side of, say, a white card that the image of a lens may be focused before we realize that the image on the card is out of focus --that is to say, before a point source images to a circle of confusion with diameter c.

Most people cannot tell the difference between a perfect point and a small circle of confusion that subtends less than about 2 arc-min (1 in 1700) at the eye.* For example, if we look at an image from a distance of 200 mm, then the circle of confusion would be approximately 200/1700 = 0.12 mm in diameter.

Suppose we wish to focus the spherical lens after the etalon by placing a white card at the 5-facet mirror and looking at the rings. We slide the lens away from the white card until we just see the rings go out of focus--the focus of the lens is in front of the card. Now we slide the lens toward the white card until we see the rings go out of focus again--the focus is now behind the white card. The total distance that we slid the lens is D which is the depth of focus.



We can derive an expression for the depth of focus D in terms of the lens focal length f, the lens aperture A (or the maximum lateral dimensions of the light at the lens), and the diameter of the circle of confusion c:

$$D = \frac{2cf}{A}$$

If we take f = 1810 mm, A = 70 mm, and c = 0.12 mm from our example above, we find that the best we can do is focus the spherical lens to within approximately 6 mm = 0.25". (I generally cannot do better than approximately 0.5", so perhaps my eyes are good to only 4 arc-min. Or perhaps using A = 70 mm is an overestimate.) We could do better by looking at the rings with a magnifying glass or by setting up a camera with high magnification, for example.

* "Optical System Design" by Rudolf Kingslake, Academic Press, 1983, p. 267

When light traveling through air encounters some other material, say, the glass in the core of our fibers or in the lenses on the analyzer table, most of the light transmits into the glass but a small percentage of the light is reflected back into the air. This happens anytime light travels from a medium of one index of refraction to a medium with a different index of refraction. Recall that air has an index of refraction $n_{air} = 1.00$ and glass has an index of refraction $n_{glass} = 1.46$.

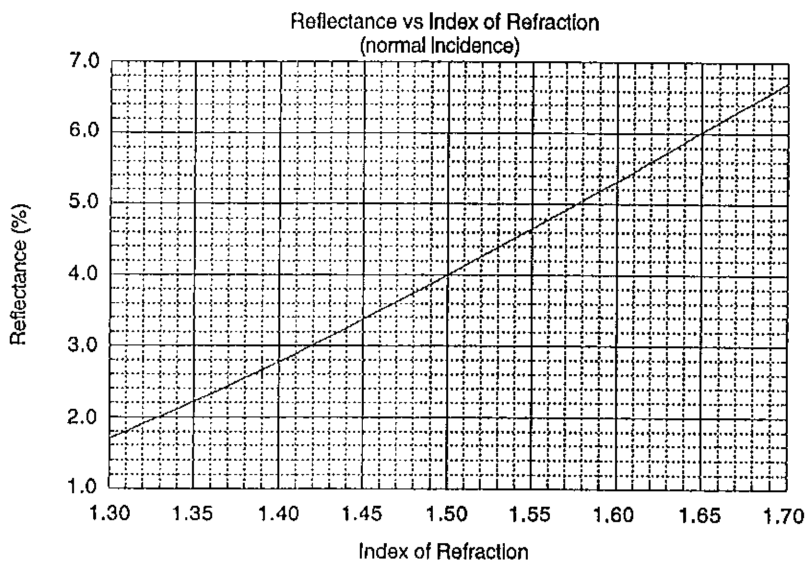
There is a simple expression to calculate what percentage of the light will be reflected when light crosses an interface at normal incidence:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

where n_1 and n_2 are the two different indices of refraction. If the light is traveling from air to another material with index n, then the equation simplifies to:

$$R = \left(\frac{n - 1}{n + 1}\right)^2$$

The following graph calculates the reflectance when light travels from air to a material with index of refraction n:

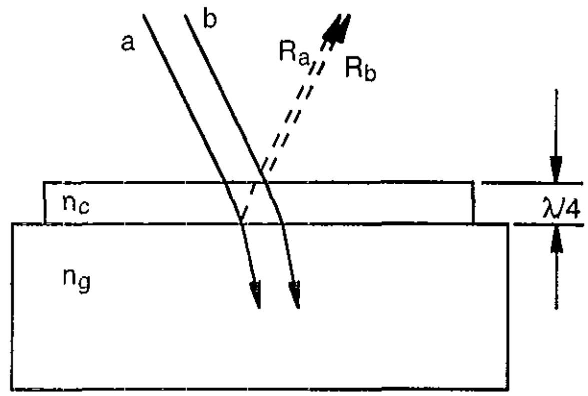


Examples:

1) For the case of glass (fused silica) with n = 1.4607, we get $R_{glass} = 0.0351 = 3.51\%$.
2) Our probe lenses are made of acrylic with index n = 1.4946, so $R_{acrylic} = 3.93\%$.

Note: The index of refraction depends upon wavelength, so be sure to use the index for the right wavelength. The values of n given above are at 532 nm. For example, the index of acrylic at 1064 nm is n = 1.4831.

The reflectivities calculated in section 8.7 may be minimized by the use of anti-reflective (AR) coatings. Reducing the amount of reflected light improves the transmission and reduces problems with crosstalk or ghost images* created by large amounts of reflected light.



The drawing shows a glass substrate with an AR coating on it. The indices of refraction of the glass and coating are n_g and n_c. Consider two rays a and b with wavelength λ positioned so that the reflection of a from the glass coincides with the reflection of b from the coating. From the equations in section 8.7, we can calculate that if

$$n_c = \sqrt{n_g}$$

then the intensities of the two reflected beams R_a and R_b will be equal. Furthermore, if the thickness of the coating is $\lambda /4$, then the two reflected beams will be out of phase and will cancel each other out.

Notes:

1) A single-layer AR coating as described here is most effective at only a single wavelength.

2) A single-layer AR coating is also most effective at only a single angle of incidence. Off-the-shelf AR coatings are made for maximum efficiency at 0° or 45° because these are the most common angles used in optical systems. Custom coatings can be made for any angle of incidence.

3) Sophisticated AR coatings are made of many thin layers of dielectric with alternating high and low indices of refraction. These coatings can be custom-designed to be very effective over a large wavelength range (broadband coatings) or at only a number of specific wavelengths or over a large range of angles.

* "Optical System Design", Rudolf Kingslake, Academic Press,1983, pp. 105-106

The emittance theorem is a simple yet powerful tool for performing certain optics calculations. In particular, the emittance theorem allows a calculation of the amount of light that may be delivered through an optical system, so long as linear optics are used. (Linear optics are things like lenses and mirrors, but not things like optical isolators, for example.) The emittance theorem relates the size of a source with the maximum range of angles of light that it emits, or the size of a receiver with the maximum range of angles of light that it accepts. The formula is:

$$E = r \times \theta = \text{constant}$$

where

E = emittance
r = radius or other linear dimension of the source or receiver
θ = maximum angle from the normal allowed by that source or receiver

Notes:
1) The emittance has dimensions of something like mm-mrad.
2) A receiver does not need to be a detector. For example, if we shine light out of a fiber, the fiber is a source. If we try to launch light into a fiber, the fiber is a receiver.
3) E = constant means that the emittance must be preserved in a linear optical system.

Example 1.
The fibers that deliver light to our analyzer tables have a diameter of 0.1 mm (radius r = 0.05 mm) and a numerical aperture NA = 0.22. From Section 2.1, we know that NA = $\sin(\theta)$ where θ is the maximum angle that light is emitted from the fiber. An NA = 0.22 yields a maximum angle of 12.7 degrees = 0.22 radians or 220 mrad. So the emittance of our 0.22 NA fibers is

$$E = 0.05 \text{ mm} \times 220 \text{ mrad} = 11 \text{ mm-mrad}$$

Example 2.
Our etalons have input stripes of approximately 0.6 mm full width or 0.3 mm half width (radius). We know (section 4.6.3) that the maximum fringe angle we can see on our streak cameras is about 6.1 mrad. This is the maximum angle of light that we can use from the etalon. If we launch light into the etalon with an angle greater than 6 mrad, then that light is lost or wasted because we cannot use it at the streak cameras. So, the acceptance emittance (in the vertical direction) of the etalon is:

$$E = 0.3 \text{ mm} \times 6.1 \text{ mrad} = 1.8 \text{ mm-mrad}$$

Note: From these two examples, we see that the acceptance emittance of the etalon is only 1.8/11 = 16% of the output emittance of the fiber. This means that it is not possible to use all of the light from our fibers on our analyzer tables, regardless of the combinations of lenses or mirrors that we try to use to improve the situation.

Section 9

Useful Information

I photocopied the following pages from:

CRC Standard Mathematical Tables, 18^{th} Edition, 1970, pp. 8 - 19.

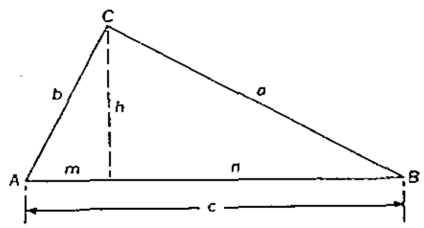
MENSURATION FORMULAE

DR. HOWARD EVES

TRIANGLES

In the following: K = area, r = radius of the inscribed circle, R = radius of the circumscribed circle.

Right Triangle



$A + B = C = 90°$

$c^2 = a^2 + b^2$ (*Pythagorean relation*)

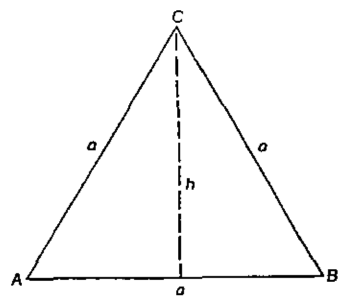
$a = \sqrt{(c + b)(c - b)}$

$K = \frac{1}{2}ab$

$r = \dfrac{ab}{a + b + c}, \quad R = \frac{1}{2}c$

$h = \dfrac{ab}{c}, \quad m = \dfrac{b^2}{c}, \quad n = \dfrac{a^2}{c}$

Equilateral Triangle



$A = B = C = 60°$

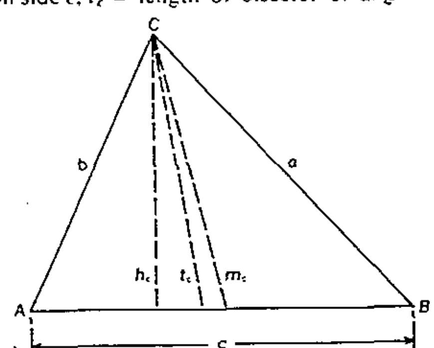
$K = \frac{1}{4}a^2\sqrt{3}$

$r = \frac{1}{6}a\sqrt{3}, \quad R = \frac{1}{3}a\sqrt{3}$

$h = \frac{1}{2}a\sqrt{3}$

General Triangle

Let $s = \frac{1}{2}(a + b + c)$, h_c = length of altitude on side c, t_c = length of bisector of angle C, m_c = length of median to side c.



$A + B + C = 180°$

$c^2 = a^2 + b^2 - 2ab\cos C$ (*law of cosines*)

$$K = \tfrac{1}{2}h_c c = \tfrac{1}{2}ab\sin C = \frac{c^2 \sin A \sin B}{2\sin C} = rs = \frac{abc}{4R} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{(Heron's formula)}$$

$$r = c\sin\frac{A}{2}\sin\frac{B}{2}\sec\frac{C}{2} = \frac{ab\sin C}{2s} = (s-c)\tan\frac{C}{2}$$

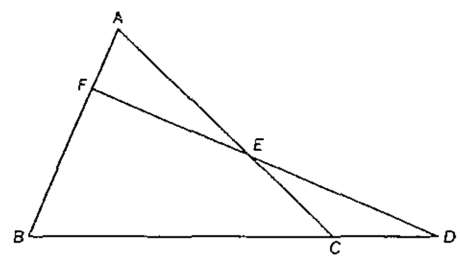
$$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} = \frac{K}{s} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

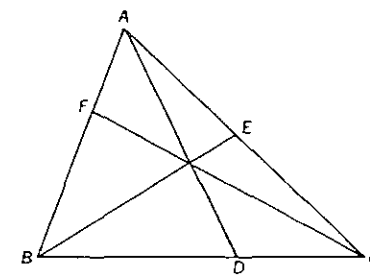
$$R = \frac{c}{2\sin C} = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}} = \frac{abc}{4K}$$

$$h_c = a\sin B = b\sin A = \frac{2K}{c}$$

$$t_c = \frac{2ab}{a+b}\cos\frac{C}{2} = \sqrt{ab\left\{1 - \frac{c^2}{(a+b)^2}\right\}}$$

$$m_c = \sqrt{\frac{a^2}{2} + \frac{b^2}{2} - \frac{c^2}{4}}$$





Menelaus' Theorem. A necessary and sufficient condition for points D, E, F on the respective side lines BC, CA, AB of a triangle ABC to be collinear is that

$$BD \cdot CE \cdot AF = -\,DC \cdot EA \cdot FB,$$

where all segments in the formula are directed segments.

Ceva's Theorem. A necessary and sufficient condition for AD, BE, CF, where D, E, F are points on the respective side lines BC, CA, AB of a triangle ABC, to be concurrent is that

$$BD \cdot CE \cdot AF = +\,DC \cdot EA \cdot FB,$$

where all segments in the formula are directed segments.

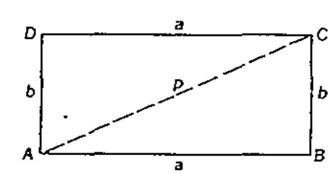
QUADRILATERALS

In the following: K = area, p and q are diagonals.

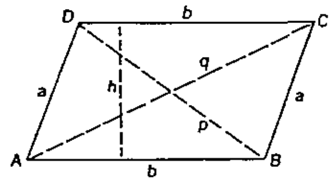
Rectangle

$A = B = C = D = 90°$

$K = ab, \quad p = \sqrt{a^2 + b^2}$



Parallelogram



$A = C, \quad B = D, \quad A + B = 180^\circ$

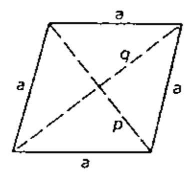
$K = bh = ab \sin A = ab \sin B$

$h = a \sin A = a \sin B$

$p = \sqrt{a^2 + b^2 - 2ab \cos A}$

$q = \sqrt{a^2 + b^2 - 2ab \cos B} = \sqrt{a^2 + b^2 + 2ab \cos A}$

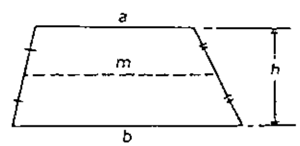
Rhombus



$p^2 + q^2 = 4a^2$

$K = \frac{1}{2} pq$

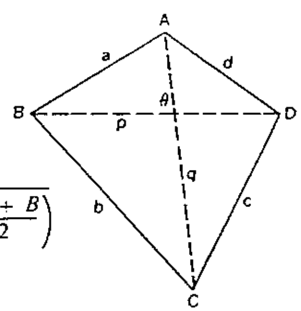
Trapezoid



$m = \frac{1}{2}(a + b)$

$K = \frac{1}{2}(a + b)h = mh$

General quadrilateral



Let $s = \frac{1}{2}(a + b + c + d)$.

$K = \frac{1}{2} pq \sin \theta$

$= \frac{1}{4}(b^2 + d^2 - a^2 - c^2) \tan \theta$

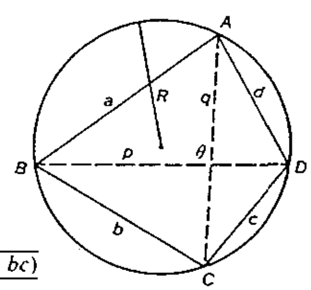
$= \frac{1}{4}\sqrt{4p^2q^2 - (b^2 + d^2 - a^2 - c^2)^2}$

(*Bretschneider's formula*)

$= \sqrt{(s - a)(s - b)(s - c)(s - d) - abcd \cos^2\left(\frac{A + B}{2}\right)}$

Theorem. The diagonals of a quadrilateral with consecutive sides a, b, c, d are perpendicular if and only if $a^2 + c^2 = b^2 + d^2$.

Cyclic Quadrilateral



Let R = radius of the circumscribed circle.

$A + C = B + D = 180^\circ$

$K = \sqrt{(s - a)(s - b)(s - c)(s - d)}$

(*Brahmagupta's formula*)

$= \frac{\sqrt{(ac + bd)(ad + bc)(ab + cd)}}{4R}$

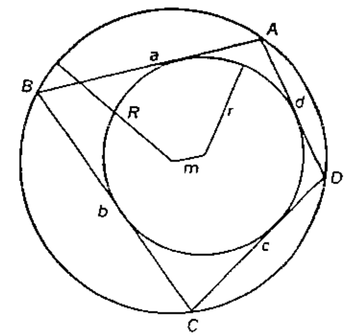
$p = \sqrt{\frac{(ac + bd)(ab + cd)}{ad + bc}}, \quad q = \sqrt{\frac{(ac + bd)(ad + bc)}{ab + cd}}$

$R = \frac{1}{4}\sqrt{\frac{(ac + bd)(ad + bc)(ab + cd)}{(s - a)(s - b)(s - c)(s - d)}}, \quad \sin \theta = \frac{2K}{ac + bd}$

Ptolemy's Theorem. A convex quadrilateral with consecutive sides a, b, c, d and diagonals p and q is cyclic if and only if $ac + bd = pq$.

Cyclic-inscriptable Quadrilateral

Let r = radius of the inscribed circle, R = radius of the circumscribed circle, m = distance between the centers of the inscribed and the circumscribed circles.



$A + C = B + D = 180^\circ$

$a + c = b + d$

$K = \sqrt{abcd}$

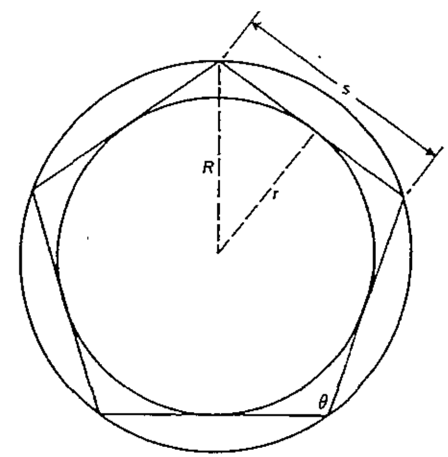
$\frac{1}{(R - m)^2} + \frac{1}{(R + m)^2} = \frac{1}{r^2}$

$r = \frac{\sqrt{abcd}}{s}$

$R = \frac{1}{2}\sqrt{\frac{(ac + bd)(ad + bc)(ab + cd)}{abcd}}$

REGULAR POLYGONS

In the following: n = number of sides, s = length of each side, p = perimeter, θ = one of the vertex angles, r = radius of the inscribed circle, R = radius of the circumscribed circle, K = area.



$\theta = \left(\frac{n - 2}{n}\right) 180^\circ$

$s = 2r \tan \frac{180^\circ}{n} = 2R \sin \frac{180^\circ}{n}$

$p = ns$

$K = \frac{1}{4} ns^2 \cot \frac{180^\circ}{n}$

$= nr^2 \tan \frac{180^\circ}{n}$

$= \frac{1}{2} nR^2 \sin \frac{360^\circ}{n}$

$r = \frac{1}{2} s \cot \frac{180^\circ}{n}, \quad R = \frac{1}{2} s \csc \frac{180^\circ}{n}$

Polygon	n	K	r	R
Triangle (equilateral)	3	$0.43301s^2$	$0.28867s$	$0.57735s$
Square	4	$1.00000s^2$	$0.50000s$	$0.70710s$
Pentagon	5	$1.72048s^2$	$0.68819s$	$0.85065s$
Hexagon	6	$2.59808s^2$	$0.86602s$	$1.00000s$
Heptagon	7	$3.63391s^2$	$1.0383s$	$1.1523s$
Octagon	8	$4.82843s^2$	$1.2071s$	$1.3065s$
Nonagon	9	$6.18182s^2$	$1.3737s$	$1.4619s$
Decagon	10	$7.69421s^2$	$1.5388s$	$1.6180s$
Undecagon	11	$9.36564s^2$	$1.7028s$	$1.7747s$
Dodecagon	12	$11.19615s^2$	$1.8660s$	$1.9318s$

If s_k denotes the side of a regular polygon of k sides inscribed in a circle of radius R, then

$$s_{2n} = \sqrt{2R^2 - R\sqrt{4R^2 - s_n^2}}.$$

If S_k denotes the side of a regular polygon of k sides circumscribed about a circle of radius r, then

$$S_{2n} = \frac{2rS_n}{2r + \sqrt{4r^2 + S_n^2}}$$

If p_k and P_k denote, respectively, the perimeters of regular polygons of k sides inscribed in and circumscribed about the same circle, then

$$P_{2n} = \frac{2p_nP_n}{p_n + P_n} \quad \text{and} \quad p_{2n} = \sqrt{p_nP_{2n}}.$$

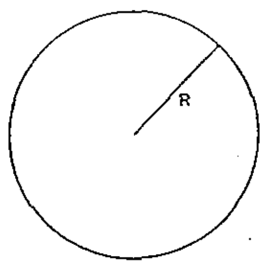
If a_k and A_k denote, respectively, the areas of regular polygons of k sides inscribed in and circumscribed about the same circle, then

$$a_{2n} = \sqrt{a_nA_n} \quad \text{and} \quad A_{2n} = \frac{2a_{2n}A_n}{a_{2n} + A_n}.$$

CIRCLES

In the following: R = radius, D = diameter, C = circumference, K = area.

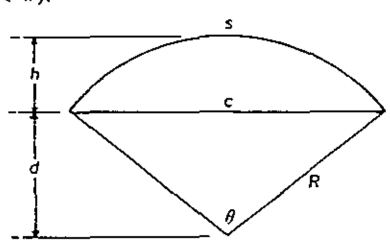
Circumference and Area of a Circle



$$C = 2\pi R = \pi D \qquad (\pi = 3.14159\cdots)$$
$$K = \pi R^2 = \tfrac{1}{4}\pi D^2 = 0.7854D^2$$
$$C = 2\sqrt{\pi K} = \frac{2K}{R}$$
$$K = \frac{C^2}{4\pi} = \tfrac{1}{2}CR$$

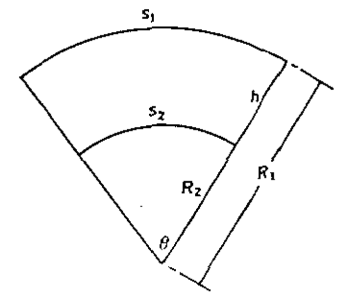
Sector and Segment of a Circle

Let the central angle θ be measured in radians ($\theta < \pi$).



$$h = R - d, \quad d = R - h$$
$$s = R\theta$$
$$d = R\cos\frac{\theta}{2} = \tfrac{1}{2}c\cot\frac{\theta}{2}$$
$$= \tfrac{1}{2}\sqrt{4R^2 - c^2}$$
$$c = 2R\sin\frac{\theta}{2} = 2d\tan\frac{\theta}{2}$$
$$= 2\sqrt{R^2 - d^2} = \sqrt{4h(2R - h)}$$
$$\theta = \frac{s}{R} = 2\,\mathrm{Cos}^{-1}\frac{d}{R} = 2\,\mathrm{Tan}^{-1}\frac{c}{2d} = 2\,\mathrm{Sin}^{-1}\frac{c}{2R}$$
$$K(\text{sector}) = \tfrac{1}{2}Rs = \tfrac{1}{2}R^2\theta$$
$$K(\text{segment}) = \tfrac{1}{2}R^2(\theta - \sin\theta) = \tfrac{1}{2}(Rs - cd) = R^2\,\mathrm{Cos}^{-1}\frac{d}{R} - d\sqrt{R^2 - d^2}$$
$$= R^2\,\mathrm{Cos}^{-1}\frac{R - h}{R} - (R - h)\sqrt{2Rh - h^2}$$

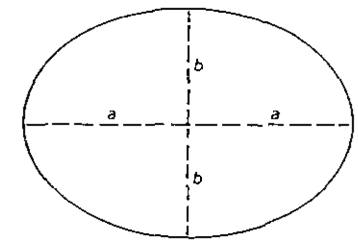
Sector of an Annulus



$$h = R_1 - R_2$$
$$K = \tfrac{1}{2}\theta(R_1 + R_2)(R_1 - R_2)$$
$$= \tfrac{1}{2}\theta h(R_1 + R_2)$$
$$= \tfrac{1}{2}h(s_1 + s_2)$$

CONIC SECTIONS

Ellipse



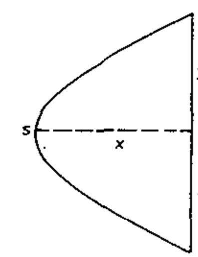
Let p = circumference, K = area

$$p = 2\pi\sqrt{\frac{a^2 + b^2}{2}} \quad \text{(approximately)}$$

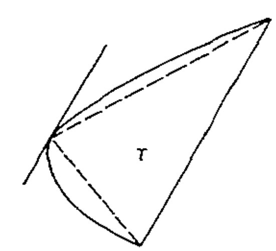
$= 4aE$ (exactly) See table of elliptic integral for E, using $k = \sqrt{a^2 - b^2}/a$.

$$K = \pi ab$$

Parabolic Segment



$$s = \sqrt{4x^2 + y^2} + \frac{y^2}{2x}\log_e\left[\frac{2x + \sqrt{4x^2 + y^2}}{y}\right]$$



K (right segment) $= \tfrac{2}{3}xy$

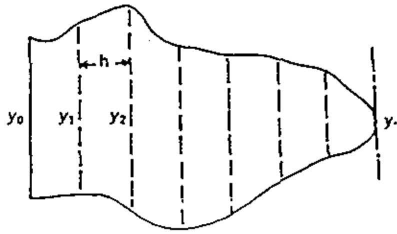
K (oblique segment) $= \tfrac{4}{3}T$, where T is the area of the triangle with base along the chord of the segment and with opposite vertex at the point on the parabola at which the tangent to the parabola is parallel to the chord of the segment.

CAVALIERI'S THEOREM FOR THE PLANE

If two planar areas are included between a pair of parallel lines, and if the two segments cut off by the areas on any line parallel to the including lines are equal in length, then the two planar areas are equal.

PLANAR AREAS BY APPROXIMATION

Divide the planar area K into n strips by equidistant parallel chords of lengths $y_0, y_1, y_2, \ldots, y_n$ (where y_0 and/or y_n may be zero), and let h denote the common distance between the chords. Then, approximately:



Trapezoidal Rule

$$K = h(\tfrac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \tfrac{1}{2}y_n)$$

Durand's Rule

$$K = h(\tfrac{4}{10}y_0 + \tfrac{11}{10}y_1 + y_2 + y_3 + \cdots + y_{n-2} + \tfrac{11}{10}y_{n-1} + \tfrac{4}{10}y_n)$$

Simpson's rule (n even)

$$K = \tfrac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$$

Weddle's Rule ($n = 6$)

$$K = \tfrac{3}{10}h(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6)$$

SOLIDS BOUNDED BY PLANES

In the following: S = lateral surface, T = total surface, V = volume.

Cube

Let a = length of each edge.

$T = 6a^2$, diagonal of face $= a\sqrt{2}$
$V = a^3$, diagonal of cube $= a\sqrt{3}$

Rectangular Parallelepiped (or box)

Let a, b, c be the lengths of its edges.

$T = 2(ab + bc + ca)$, $V = abc$
diagonal $= \sqrt{a^2 + b^2 + c^2}$

Prism

S = (perimeter of right section) × (lateral edge)
V = (area of right section) × (lateral edge)
= (area of base) × (altitude)

Truncated Triangular Prism

V = (area of right section) × $\frac{1}{3}$(sum of the three lateral edges)

Pyramid

S of regular pyramid = $\frac{1}{2}$(perimeter of base) × (slant height)
$V = \frac{1}{3}$(area of base) × (altitude)

Frustum of Pyramid

Let B_1 = area of lower base, B_2 = area of upper base, h = altitude.

S of regular figure = $\frac{1}{2}$(sum of perimeters of bases) × (slant height)
$V = \frac{1}{3}h(B_1 + B_2 + \sqrt{B_1 B_2})$

Prismatoid

A *prismatoid* is a polyhedron having for bases two polygons in parallel planes, and for lateral faces triangles or trapezoids with one side lying in one base, and the opposite vertex or side lying in the other base, of the polyhedron. Let B_1 = area of lower base, M = area of midsection, B_2 = area of upper base, h = altitude.

$$V = \tfrac{1}{6}h(B_1 + 4M + B_2) \quad \text{(the } \textit{prismoidal formula}\text{)}$$

Note: Since cubes, rectangular parallelepipeds, prisms, pyramids, and frustums of pyramids are all examples of prismatoids, the formula for the volume of a prismatoid subsumes most of the above volume formulae.

Regular Polyhedra

Let v = number of vertices, e = number of edges, f = number of faces, α = each dihedral angle, a = length of each edge, r = radius of the inscribed sphere, R = radius of the circumscribed sphere, A = area of each face, T = total area, V = volume.

$v - e + f = 2$ (the *Euler-Descartes formula*—actually holds for *any* convex polyhedron)
$T = fA$
$V = \frac{1}{3}rfA = \frac{1}{3}rT$

Name	Nature of Surface	T	V
Tetrahedron	4 equilateral triangles	$1.73205a^2$	$0.11785a^3$
Hexahedron (cube)	6 squares	$6.00000a^2$	$1.00000a^3$
Octahedron	8 equilateral triangles	$3.46410a^2$	$0.47140a^3$
Dodecahedron	12 regular pentagons	$20.64573a^2$	$7.66312a^3$
Icosahedron	20 equilateral triangles	$8.66025a^2$	$2.18170a^3$

Name	v	e	f	α	a	r
Tetrahedron	4	6	4	70° 32′	$1.633R$	$0.333R$
Hexahedron	8	12	6	90°	$1.155R$	$0.577R$
Octahedron	6	12	8	109° 28′	$1.414R$	$0.577R$
Dodecahedron	20	30	12	116° 34′	$0.714R$	$0.795R$
Icosahedron	12	30	20	138° 11′	$1.051R$	$0.795R$

Name	A	r	R	V
Tetrahedron	$\frac{1}{4}a^2\sqrt{3}$	$\frac{1}{12}a\sqrt{6}$	$\frac{1}{4}a\sqrt{6}$	$\frac{1}{12}a^3\sqrt{2}$
Hexahedron	a^2	$\frac{1}{2}a$	$\frac{1}{2}a\sqrt{3}$	a^3
Octahedron	$\frac{1}{4}a^2\sqrt{3}$	$\frac{1}{6}a\sqrt{6}$	$\frac{1}{2}a\sqrt{2}$	$\frac{1}{3}a^3\sqrt{2}$
Dodecahedron	$\frac{1}{4}a^2\sqrt{25 + 10\sqrt{5}}$	$\frac{1}{20}a\sqrt{250 + 110\sqrt{5}}$	$\frac{1}{4}a(\sqrt{15} + \sqrt{3})$	$\frac{1}{4}a^3(15 + 7\sqrt{5})$
Icosahedron	$\frac{1}{4}a^2\sqrt{3}$	$\frac{1}{12}a\sqrt{42 + 18\sqrt{5}}$	$\frac{1}{4}a\sqrt{10 + 2\sqrt{5}}$	$\frac{5}{12}a^3(3 + \sqrt{5})$

CYLINDERS AND CONES

In the following: B_1 = area of lower base, B_2 = area of upper base, h = altitude, S = lateral surface, T = total surface, V = volume.

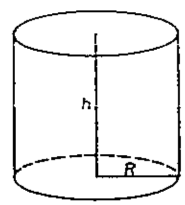
Cylinder

S = (perimeter of right section) × (lateral edge)
V = (area of right section) × (lateral edge)

Right Circular Cylinder

Let R = radius of base.

$$S = 2\pi Rh, \quad T = 2\pi R(R + h), \quad V = \pi R^2 h$$



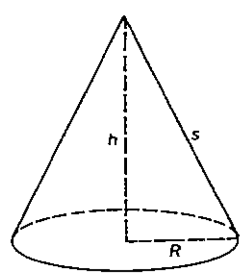
Cone

$$V = \tfrac{1}{3}B_1 h$$

Right Circular Cone

Let R = radius of base, s = slant height.

$$s = \sqrt{R^2 + h^2}$$
$$S = \pi Rs = \pi R\sqrt{R^2 + h^2}$$
$$T = \pi R(R + s) = \pi R(R + \sqrt{R^2 + h^2})$$
$$V = \tfrac{1}{3}\pi R^2 h$$



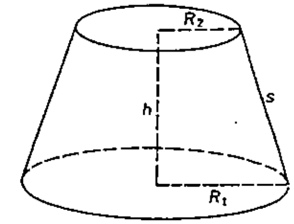
Frustum of Cone

$$V = \tfrac{1}{3}h(B_1 + B_2 + \sqrt{B_1 B_2})$$

Frustum of Right Circular Cone

Let R_1 = radius of lower base, R_2 = radius of upper base, s = slant height.

$$s = \sqrt{(R_1 - R_2)^2 + h^2}$$
$$S = \pi(R_1 + R_2)s$$
$$= \pi(R_1 + R_2)\sqrt{(R_1 - R_2)^2 + h^2}$$
$$T = \pi[R_1^2 + R_2^2 + (R_1 + R_2)s]$$
$$= \pi[R_1^2 + R_2^2 + (R_1 + R_2)\sqrt{(R_1 - R_2)^2 + h^2}]$$
$$V = \tfrac{1}{3}\pi h(R_1^2 + R_2^2 + R_1R_2)$$



SPHERICAL FIGURES

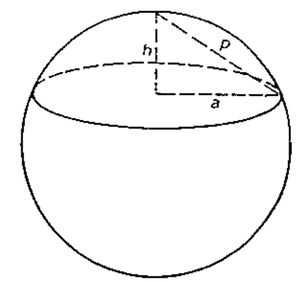
In the following: R = radius of sphere, D = diameter of sphere, S = surface area, V = volume.

Sphere

$$D = 2R$$
$$S = 4\pi R^2 = \pi D^2 = 12.57R^2$$
$$V = \tfrac{4}{3}\pi R^3 = \tfrac{1}{6}\pi D^3 = 4.189R^3$$

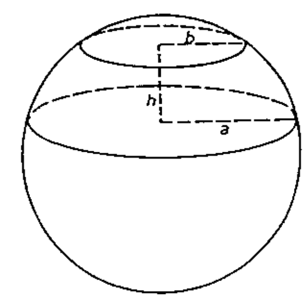
Zone and Segment of One Base

$$S = 2\pi Rh = \pi Dh = \pi p^2$$
$$V = \tfrac{1}{3}\pi h^2(3R - h) = \tfrac{1}{6}\pi h(3a^2 + h^2)$$



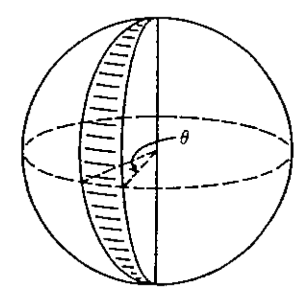
Zone and Segment of Two Bases

$$S = 2\pi Rh = \pi Dh$$
$$V = \tfrac{1}{6}\pi h(3a^2 + 3b^2 + h^2)$$



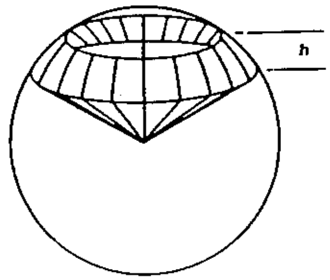
Lune

$S = 2R^2\theta$, θ in radians



Spherical Sector

$$V = \tfrac{2}{3}\pi R^2 h = \tfrac{1}{6}\pi D^2 h$$



Spherical Triangle and Polygon

Let A, B, C be the angles, in radians, of the triangle; let θ = sum of angles, in radians, of a spherical polygon of n sides.

$$S = (A + B + C - \pi) R^2$$
$$S = [\theta - (n - 2)\pi] R^2$$

SPHEROIDS

Ellipsoid

Let a, b, c be the lengths of the semiaxes.

$$V = \tfrac{4}{3}\pi abc$$

Oblate Spheroid

An *oblate spheroid* is formed by the rotation of an ellipse about its minor axis. Let a and b be the major and minor semiaxes, respectively, and ϵ the eccentricity, of the revolving ellipse.

$$S = 2\pi a^2 + \pi \frac{b^2}{\epsilon} \log_e \frac{1+\epsilon}{1-\epsilon}$$
$$V = \tfrac{4}{3}\pi a^2 b$$

Prolate Spheroid

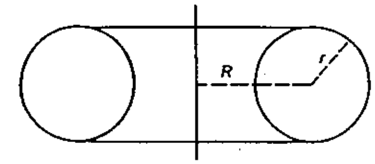
A *prolate spheroid* is formed by the rotation of an ellipse about its major axis. Let a and b be the major and minor semiaxes, respectively, and ϵ the eccentricity, of the revolving ellipse.

$$S = 2\pi b^2 + 2\pi \frac{ab}{\epsilon} \sin^{-1} \epsilon$$
$$V = \tfrac{4}{3}\pi ab^2$$

CIRCULAR TORUS

A *circular torus* is formed by the rotation of a circle about an axis in the plane of the circle and not cutting the circle. Let r be the radius of the revolving circle and let R be the distance of its center from the axis of rotation.

$$S = 4\pi^2 Rr$$
$$V = 2\pi^2 Rr^2$$



PAPPUS-GULDINUS THEOREMS

1. If a planar arc be revolved about an axis in its plane, but not cutting the arc, the area of the surface of revolution so formed is equal to the product of the length of the arc and the length of the path traced by the centroid of the arc.
2. If a planar area be revolved about an axis in its plane, but not intersecting the area, the volume of the solid of revolution so formed is equal to the product of the area and the length of the path traced by the centroid of the area.

CAVALIERI'S THEOREM FOR SPACE

If two solids are included between a pair of parallel planes, and if the two sections cut by them on any plane parallel to the including planes are equal in area, then the volumes of the solids are equal.

GENERAL PRISMATOID

A *general prismatoid* is a solid such that the area A_y of any section parallel to and distant y from a fixed plane can be expressed as a polynomial in y of degreee not higher than the third. That is,

$$A_y = ay^3 + by^2 + cy + d,$$

where a, b, c, d are constants which may be positive, zero, or negative. Let B_1 = area of lower base, M = area of midsection, B_2 = area of upper base, h = altitude.

$$V = \tfrac{1}{6}h(B_1 + 4M + B_2)$$

Note. All prismatoids, cylinders, cones, spheres, spheroids, and many other solids are general prismatoids.

CENTROIDS

If a geometrical figure possesses a center of symmetry, that point is the centroid of the figure.

If a geometrical figure possesses an axis of symmetry, the centroid of the figure lies on that axis.

I photocopied the following pages from:

CRC Standard Mathematical Tables, 18th Edition, 1970, pp. 190 - 191.

Pythagorean relations

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad 1 + \tan^2\alpha = \sec^2\alpha, \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}$$
$$\cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$
$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$
$$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

Double-angle relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}, \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

Multiple-angle relations

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$
$$\sin 4\alpha = 4\sin\alpha\cos\alpha - 8\sin^3\alpha\cos\alpha$$
$$\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$$
$$\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$$
$$\cos 5\alpha = 16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$$
$$\sin 6\alpha = 32\cos^5\alpha\sin\alpha - 32\cos^3\alpha\sin\alpha + 6\cos\alpha\sin\alpha$$
$$\cos 6\alpha = 32\cos^6\alpha - 48\cos^4\alpha + 18\cos^2\alpha - 1$$
$$\sin n\alpha = 2\sin(n-1)\alpha\cos\alpha - \sin(n-2)\alpha$$
$$\cos n\alpha = 2\cos(n-1)\alpha\cos\alpha - \cos(n-2)\alpha$$
$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$
$$\tan 4\alpha = \frac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$$
$$\tan n\alpha = \frac{\tan(n-1)\alpha + \tan\alpha}{1 - \tan(n-1)\alpha\tan\alpha}$$

Function-product relations

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\sin(\alpha + \beta) - \tfrac{1}{2}\sin(\alpha - \beta)$$

Function-sum and function-difference relations

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\tan\alpha + \tan\beta = \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta}, \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha + \cot\beta = \frac{\sin(\alpha + \beta)}{\sin\alpha\sin\beta}, \qquad \cot\alpha - \cot\beta = \frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta}$$
$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\tan\frac{1}{2}(\alpha + \beta)}{\tan\frac{1}{2}(\alpha - \beta)}, \qquad \frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\tfrac{1}{2}(\beta - \alpha)$$
$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha + \beta), \qquad \frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha - \beta)$$

Half-angle relations

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}, \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$
$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} = \frac{1 + \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 - \cos\alpha}$$

Power relations

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha), \qquad \sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha)$$
$$\sin^4\alpha = \tfrac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$$
$$\cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha), \qquad \cos^3\alpha = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha)$$
$$\cos^4\alpha = \tfrac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$$
$$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}, \qquad \cot^2\alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$$

Exponential relations (α in radians). Euler's equation

$$e^{i\alpha} = \cos\alpha + i\sin\alpha, \qquad i = \sqrt{-1}$$
$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}, \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$
$$\tan\alpha = -i\left(\frac{e^{i\alpha} - e^{-i\alpha}}{e^{i\alpha} + e^{-i\alpha}}\right) = -i\left(\frac{e^{2i\alpha} - 1}{e^{2i\alpha} + 1}\right)$$

I photocopied the following pages from:

CRC Standard Mathematical Tables, 18^{th} Edition, 1970, pp. 454 - 457.

SERIES

The expression in parentheses following certain of the series indicates the region of convergence. If not otherwise indicated it is to be understood that the series converges for all finite values of x.

BINOMIAL

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots \quad (y^2 < x^2)$$

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)x^2}{2!} \pm \frac{n(n-1)(n-2)x^3}{3!} + \cdots \text{etc.} \quad (x^2 < 1)$$

$$(1 \pm x)^{-n} = 1 \mp nx + \frac{n(n+1)x^2}{2!} \mp \frac{n(n+1)(n+2)x^3}{3!} + \cdots \text{etc.} \quad (x^2 < 1)$$

$$(1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + x^4 \mp x^5 + \cdots \quad (x^2 < 1)$$

$$(1 \pm x)^{-2} = 1 \mp 2x + 3x^2 \mp 4x^3 + 5x^4 \mp 6x^5 + \cdots \quad (x^2 < 1)$$

REVERSION OF SERIES

Let a series be represented by

$$y = a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + \cdots \quad (a_1 \neq 0)$$

to find the coefficients of the series

$$x = A_1y + A_2y^2 + A_3y^3 + A_4y^4 + \cdots$$

$$A_1 = \frac{1}{a_1} \qquad A_2 = -\frac{a_2}{a_1^3} \qquad A_3 = \frac{1}{a_1^5}(2a_2^2 - a_1a_3)$$

$$A_4 = \frac{1}{a_1^7}(5a_1a_2a_3 - a_1^2a_4 - 5a_2^3)$$

$$A_5 = \frac{1}{a_1^9}(6a_1^2a_2a_4 + 3a_1^2a_3^2 + 14a_2^4 - a_1^3a_5 - 21a_1a_2^2a_3)$$

$$A_6 = \frac{1}{a_1^{11}}(7a_1^3a_2a_5 + 7a_1^3a_3a_4 + 84a_1a_2^3a_3 - a_1^4a_6 - 28a_1^2a_2^2a_4 - 28a_1^2a_2a_3^2 - 42a_2^5)$$

$$\begin{aligned} A_7 = \frac{1}{a_1^{13}}(&8a_1^4a_2a_6 + 8a_1^4a_3a_5 + 4a_1^4a_4^2 + 120a_1^2a_2^3a_4 \\ &+ 180a_1^2a_2^2a_3^2 + 132a_2^6 - a_1^5a_7 \\ &- 36a_1^3a_2^2a_5 - 72a_1^3a_2a_3a_4 - 12a_1^3a_3^3 - 330a_1a_2^4a_3) \end{aligned}$$

TAYLOR

1. $$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots + \frac{(x-a)^n}{n!}f^{(n)}(a) + \cdots \text{(Taylor's Series)}$$

(Increment form)

2. $$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots$$
$$= f(h) + xf'(h) + \frac{x^2}{2!}f''(h) + \frac{x^3}{3!}f'''(h) + \cdots$$

3. If $f(x)$ is a function possessing derivatives of all orders throughout the interval $a \leqq x \leqq b$, then there is a value X, with $a < X < b$, such that

$$f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \cdots + \frac{(b-a)^{n-1}}{(n-1)!}f^{(n-1)}(a) + \frac{(b-a)^n}{n!}f^{(n)}(X)$$

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \cdots + \frac{h^{n-1}}{(n-1)!}f^{(n-1)}(a) + \frac{h^n}{n!}f^{(n)}(a+\theta h), \quad b = a+h,\ 0 < \theta < 1.$$

or

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots + (x-a)^{n-1}\frac{f^{(n-1)}(a)}{(n-1)!} + R_n,$$

where

$$R_n = \frac{f^{(n)}[a + \theta\cdot(x-a)]}{n!}(x-a)^n, \quad 0 < \theta < 1.$$

The above forms are known as Taylor's series with the remainder term.

4. *Taylor's series for a function of two variables*

If $$\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)f(x,y) = h\frac{\partial f(x,y)}{\partial x} + k\frac{\partial f(x,y)}{\partial y};$$

$$\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(x,y) = h^2\frac{\partial^2 f(x,y)}{\partial x^2} + 2hk\frac{\partial^2 f(x,y)}{\partial x\,\partial y} + k^2\frac{\partial^2 f(x,y)}{\partial y^2}$$

etc., and if $h\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^n f(x,y)\Big|_{\substack{x=a\\y=b}}$ with the bar and subscripts means that after differentiation we are to replace x by a and y by b,

$$f(a+h, b+k) = f(a,b) + \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)f(x,y)\Big|_{\substack{x=a\\y=b}} + \cdots + \frac{1}{n!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^n f(x,y)\Big|_{\substack{x=a\\y=b}} + \cdots$$

MACLAURIN

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots + x^{n-1}\frac{f^{(n-1)}(0)}{(n-1)!} + R_n,$$

where

$$R_n = \frac{x^n f^{(n)}(\theta x)}{n!}, \quad 0 < \theta < 1.$$

EXPONENTIAL

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \qquad \text{(all real values of } x\text{)}$$

$$a^x = 1 + x\log_e a + \frac{(x\log_e a)^2}{2!} + \frac{(x\log_e a)^3}{3!} + \cdots$$

$$e^x = e^a\left[1 + (x - a) + \frac{(x-a)^2}{2!} + \frac{(x-a)^3}{3!} + \cdots\right]$$

LOGARITHMIC

$$\log_e x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \cdots \qquad (x > \tfrac{1}{2})$$

$$\log_e x = (x - 1) - \tfrac{1}{2}(x-1)^2 + \tfrac{1}{3}(x-1)^3 - \cdots \qquad (2 \geq x > 0)$$

$$\log_e x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right] \qquad (x > 0)$$

$$\log_e(1 + x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \cdots \qquad (-1 < x < 1)$$

$$\log_e(n + 1) - \log_e(n - 1) = 2\left[\frac{1}{n} + \frac{1}{3n^3} + \frac{1}{5n^5} + \cdots\right]$$

$$\log_e(a + x) = \log_e a + 2\left[\frac{x}{2a+x} + \frac{1}{3}\left(\frac{x}{2a+x}\right)^3 + \frac{1}{5}\left(\frac{x}{2a+x}\right)^5 + \cdots\right]$$

$$(a > 0,\ -a < x < +\infty)$$

$$\log_e\frac{1+x}{1-x} = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n-1}}{2n-1} + \cdots\right], \qquad -1 < x < 1$$

$$\log_e x = \log_e a + \frac{(x-a)}{a} - \frac{(x-a)^2}{2a^2} + \frac{(x-a)^3}{3a^3} - + \cdots, \qquad 0 < x \leqq 2a$$

TRIGONOMETRIC

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad \text{(all real values of } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad \text{(all real values of } x\text{)}$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n}{(2n)!}x^{2n-1} + \cdots.$$

$$\left[x^2 < \frac{\pi^2}{4}, \text{ and } B_n \text{ represents the } n\text{'th Bernoulli number.}\right]$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} - \cdots - \frac{2^{2n}B_n}{(2n)!}x^{2n-1} - \cdots.$$

$$[x^2 < \pi^2, \text{ and } B_n \text{ represents the } n\text{'th Bernoulli number.}]$$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \cdots + \frac{E_n x^{2n}}{(2n)!} + \cdots.$$

$$\left[x^2 < \frac{\pi^2}{4}, \text{ and } E_n \text{ represents the } n\text{'th Euler number.}\right]$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7}{360}x^3 + \frac{31}{15{,}120}x^5 + \frac{127}{604{,}800}x^7 + \cdots$$

$$+ \frac{2(2^{2n-1}-1)}{(2n)!}B_n x^{2n-1} + \cdots,$$

$$[x^2 < \pi^2, \text{ and } B_n \text{ represents } n\text{'th Bernoulli number.}]$$

$$\sin x = x\left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{2^2\pi^2}\right)\left(1 - \frac{x^2}{3^2\pi^2}\right)\cdots \qquad (x^2 < \infty)$$

$$\cos x = \left(1 - \frac{4x^2}{\pi^2}\right)\left(1 - \frac{4x^2}{3^2\pi^2}\right)\left(1 - \frac{4x^2}{5^2\pi^2}\right)\cdots \qquad (x^2 < \infty)$$

$$\sin^{-1}x = x + \frac{x^3}{2\cdot 3} + \frac{1\cdot 3}{2\cdot 4\cdot 5}x^5 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7}x^7 + \cdots \qquad \left(x^2 < 1,\ -\frac{\pi}{2} < \sin^{-1}x < \frac{\pi}{2}\right)$$

$$\cos^{-1}x = \frac{\pi}{2} - \left(x + \frac{x^3}{2\cdot 3} + \frac{1\cdot 3}{2\cdot 4\cdot 5}x^5 + \frac{1\cdot 3\cdot 5 x^7}{2\cdot 4\cdot 6\cdot 7} + \cdots\right) \qquad (x^2 < 1, 0 < \cos^{-1}x < \pi)$$

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad (x^2 < 1)$$

$$\tan^{-1}x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^2} - \frac{1}{5x^5} + \frac{1}{7x^7} - \cdots \qquad (x > 1)$$

$$\tan^{-1}x = -\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^2} - \frac{1}{5x^5} + \frac{1}{7x^7} - \cdots \qquad (x < -1)$$

$$\cot^{-1}x = \frac{\pi}{2} - x + \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7} - \cdots \qquad (x^2 < 1)$$

$$\log_e \sin x = \log_e x - \frac{x^2}{6} - \frac{x^4}{180} - \frac{x^6}{2835} - \cdots \qquad (x^2 < \pi^2)$$

$$\log_e \cos x = -\frac{x^2}{2} - \frac{x^4}{12} - \frac{x^6}{45} - \frac{17x^8}{2520} - \cdots \qquad \left(x^2 < \frac{\pi^2}{4}\right)$$

$$\log_e \tan x = \log_e x + \frac{x^2}{3} + \frac{7x^4}{90} + \frac{62x^6}{2835} + \cdots \qquad \left(x^2 < \frac{\pi^2}{4}\right)$$

$$e^{\sin x} = 1 + x + \frac{x^2}{2!} - \frac{3x^4}{4!} - \frac{8x^5}{5!} - \frac{3x^6}{6!} + \frac{56x^7}{7!} + \cdots$$

$$e^{\cos x} = e\left(1 - \frac{x^2}{2!} + \frac{4x^4}{4!} - \frac{31x^6}{6!} + \cdots\right)$$

$$e^{\tan x} = 1 + x + \frac{x^2}{2!} + \frac{3x^3}{3!} + \frac{9x^4}{4!} + \frac{37x^5}{5!} + \cdots \qquad \left(x^2 < \frac{\pi^2}{4}\right)$$

$$\sin x = \sin a + (x - a)\cos a - \frac{(x-a)^2}{2!}\sin a$$

$$- \frac{(x-a)^3}{3!}\cos a + \frac{(x-a)^4}{4!}\sin a + \cdots$$

I photocopied the following periodic table from Battelle-Northwest, who received permission to reproduce it from the Sargent-Welch Scientific Company.

This is my favorite version of the periodic table. In addition to the usual information such as atomic number and weight, it also includes the melting point, boiling point, and the density for each element.

Material	Examples	Density (g/cc)	Sound Speed (mm/μs)	Index of Refraction
Aluminum		2.7	6.42	
Beryllium		1.87	12.9	
Brass		8.6	4.7	
Copper, rolled		8.93	5.01	
Lead, rolled		11.4	1.96	
Steel, mild		7.85	5.96	
Steel, SST		7.9	5.79	
Tantalum		16.69	4.14	
Tin, rolled		7.3	3.32	
Tungsten		19.3	5.22	
Zinc		7.1	4.21	
Acrylic				
Plexiglass				
	Nested lenses			
	532 nm	1.18	2.68	1.4946
	1064 nm			1.4831
Fused silica				
	Fiber cores			
	Glass etalons			
	532 nm	2.2	5.97	1.46071
Lexan				
Polycarbonate				
	Windows			
	??? nm	1.2		1.6
Air		1.3×10^{-3}	0.33	1.003

(Most of this information is from "CRC Handbook of Chemistry and Physics", 62nd Edition, 1981-1982, p. E-45.)

Section 10

Useful Formulas

10.1 Useful Formulas

Velocity calculation

$$v = f_c \left(\frac{D_i^2 - D_1^2}{D_2^2 - D_1^2} + n \right)$$

where the fringe constant is

$$f_c = \frac{c\lambda}{4\left[H + T\left(n - \lambda \frac{dn}{d\lambda}\right)\right]}$$

$$f_c[mm/\mu s] = \frac{39.88}{H[mm] + 1.485 * T[mm]}$$

Finesse F

$$F = \frac{D_{j+1}^2 - D_j^2}{4\, D_j\, \Delta D_j} = \frac{D_j}{4j\, \Delta D_j}$$

where

$$\Delta D_j = \text{FWHM}$$

Fringe angles

$$\theta_j = \sqrt{\frac{j\lambda}{H + \frac{T}{n}}}$$

$$n = 1.46071$$